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15/ENG06/001
MECHANICAL ENGINEERING.

1. $dy/dx + 3y = e^{-2t}$ at $t=0$; $y=2$

$$sY(s) - Y(0) + 3Y(s) = \frac{1}{s+2}$$

$$sY(s) + 3Y(s) - 2 = \frac{1}{s+2}$$

$$Y(s)(s+3) = \frac{1+2(s+2)}{s+2}$$

$$Y(s) = \frac{2s+5}{(s+2)(s+3)}$$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$$

$$2s+5 = A(s+3) + B(s+2)$$

$$2s+5 = As + 3A + Bs + 2B$$

$$A+B=2 \quad \text{—①} \times 3$$

$$3A+2B=5 \quad \text{—②}$$

$$3A+3B=6$$

$$3A+2B=5$$

$$B=1$$

$$A+1=2$$

$$A=1$$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$$

$$Y(s) = e^{-2t} + e^{-3t}$$

2. $3\,dy/dt - 6y = \sin 2t$ given that $t=0$; $y=1$

$$3sY(s) - 3Y(0) - 6Y(s) = \frac{2}{s^2+2^2}$$

$$Y(s)(3s-6) = \frac{2}{(s+2)^2} + 3$$

$$Y(s) = \frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{(3s-6)}$$

$$2+3(s+2)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$$

$$2+3s^2+12s+12 = 3As^2 - 12A + 3Bs - 6B + Cs^2 + 4Cs + 4C$$

$$3A+C=3 \quad \text{—①}$$

$$3B+4C=12 \quad \text{—②}$$

$$-12A-6B+4C=1 \quad \text{—③}$$

$$3A = 3-C$$

$A = \frac{3-C}{3}$

$3B + 4C = 12$

$-12\left(\frac{3-C}{3}\right) - 6B + 4C = 17$

$-6B + 8C = 26$

$-18B - 24B = -72$

$-48C = -150$

$C = 25/8$

$3B = 12 - 4(25/8)$

$B = -1/6$

$3A = 3 - C$

$A = -1/24$

$\frac{2+3(S+2)^2}{(S+2)^2(3S-6)} = \frac{-1/24}{(S+2)} - \frac{1/6}{(S+2)^2} + \frac{25/8}{(3S-6)}$

$y(t) = -1/24\, e^{-2t} - 1/6\, t e^{-4t} + 24/24\, e^{3t}$

3. $dy/dt - 4y = 8$ given that $t=0$, $y=2$.

$sY(s) - Y(0) - 4Y(s) = 8/s$

$sY(s) - 4Y(s) - Y(0) = 8/s$

$Y(s)(s-4) = \frac{8+2s}{s}$

$Y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$8 + 2s = A(s-4) + Bs$

$A + B = 2$

$-4A = 8$

$A = -2$

$B = 2 + 2 = 4$

$L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{2}{s} + \frac{4}{s-4}\right\}$

$y = -2 + 4e^{4t}$

4) $d^2y/dt^2 - 2dy/dt + 5y = e^{2t}$ $t=0$ $y=2$ $\dot{y}=1$

$s^2Y(s) - 5Y(s) - Y(0) - 2sY(s) + 2Y(0) + 3Y(s) = \frac{1}{s-2}$

$s^2Y(s) - 2sY(s) + 5Y(s) - 2s - 1 + 4 = \frac{1}{s-2}$

$Y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 3$

$Y(s)(s^2-2s+5) = \frac{2s-3}{(s-2)}$

$Y(s) = \frac{1+2s^2-s+6}{(s-2)(s^2-2s+5)} = \frac{A}{(s-2)} + \frac{Bs+C}{s^2-2s+5}$

$2s^2-7s+7 = A(s^2-2s+5) + (Bs+C)(s-2)$

$= As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$A+B=2$ ——①

$-2A-2B+C=-7$ ——②

$B = 2-A$

$-2A-2(2-A)+C=-7$

$-2A-4+2A+C=-7$

$C=-3$

$3A-2(-3)=7$

$A = 1/5$

$A+B=2$

$B = 9/5$

$y(t) = L^{-1}\left\{\frac{1/5}{(s-2)} + \frac{9/5s}{(s^2-2s+5)} - \frac{3}{s^2-2s+5}\right\}$

$= L^{-1}\left\{\frac{1/5}{(s-2)} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2+2^2} + \frac{1}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right] - \frac{3}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right\}$

$= \frac{1}{5}e^{2t} + \frac{9}{5}\left[e^t\cos 2t + \frac{1}{2}e^t\sin 2t\right] - \frac{3}{2}(e^t\sin 2t)$

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5 $d^2y/dt^2 - 6\,dy/dt + 8y = e^{3t}$ at $t=0$, $y=0$ $\dot{y}=2$

$s^2Y(s) - sy(0) - y'(0) - 6sY(s) + 6y(0) + 8Y(s) = \frac{1}{s-3}$

$Y(s)(s^2-6s+8) = \frac{1}{s-3} + 2$

$Y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{s-3} + \frac{Bs+C}{(s^2-6s+8)}$

$2s-5 = A(s^2-6s+8) + (Bs-C)(s-3)$

$= As^2 - 6As + 8A + Bs^2 - 3Bs + Cs - 3C$

$A+B=0$

$-6A-3B+C=2$

$8A-3C=-3$

$B=-A$

$-6A+3A+C=2$

$-3A+C=2$

$8A-3C=-5$

$9A - 3C = -6$

$8A - 3C = -5$

$A = -1$

$B = 1$

$C = 2 - 3$

$C = -1$

$\frac{2s-5}{(s+3)(s-3)}$ $Y(s) = L^{-1}\left\{\frac{-1}{s-3} + \frac{s-1}{(s^2-6s+8)}\right\}$

$= L^{-1}\left\{\frac{-1}{s-3} + \frac{s-1}{(s-2)(s-4)}\right\}$

$\frac{s-1}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$s-1 = A(s-4) + B(s-2)$

$s-1 = As - 4A + Bs - 2B$

$A + B = 1$

$-4A + -2B = -1$

$-4A - 4B = -4$

$-4A - 2B = -1$

$-2B = -3$

$B = 3/2$

$A = -1/2$

$Y(s) = L^{-1}\left\{\frac{-1}{s-3} + \left(\frac{-1/2}{(s-2)} + \frac{3/2}{(s-4)}\right)\right\}$

$y = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$

$y = \frac{1}{2}\left[2e^{3t} + e^{2t} - 3e^{4t}\right]$