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1 $\frac{dy}{dt} + 3y = e^{-2t}$ given at $t = 0$ $y = 2$

$y(0) = 2$

$y^{(1)} + 3y = e^{-2t}$

$Sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$Sy(s) - 2 + 3y(s) = \frac{1}{s+2}$

$y(s)[s+3] = \frac{1}{s+2} + \frac{2}{1} \cdot \frac{1+2(s+2)}{s+2} = \frac{1+2s+4}{s+2}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$A = \left.\frac{1+2s+4}{s+3}\right|_{s=-2} = \frac{1+2(-2)+4}{-2+3} = 1$

$B = \left.\frac{1+2s+4}{s+2}\right|_{s=-3} = \frac{1+2(-3)+4}{-3+2} = 1$

$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = e^{-2t} + e^{-3t}$

2 Question 2

2 $3\frac{dy}{dt} - 6y = \sin 2t$ at $t = 0$ $y = 1$

$y(s) = 1$

$3y^{(1)} - 6y = \sin 2t$

$3[Sy(s) - y(0)] - 6y(s) = \frac{2}{s^2+4}$

$3Sy(s) - 3y(s) - 6y(s) = \frac{2}{s^2+4}$

$Sy(s)[3s-6] = \frac{2}{s^2+4} + \frac{3}{1} = \frac{2+3(s^2+4)}{s^2+4}$

$$Y(s)[3s-6] = \frac{2+3s^2+12}{(s^2+4)} = \frac{3s^2+14}{(s^2+4)}$$

$$Y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{(s^2+4)} + \frac{C}{3s-6}$$

$$C: \left.\frac{3s^2+14}{s^2+14}\right|_{s=2} = \frac{3(2)^2+14}{2^2+4} = \frac{13}{4}$$

$$3s^2+14 = (A+Bs)(3s-6) + C(s^2+4)$$
$$3s^2+14 = 3As-6A+3Bs^2-6Bs+Cs^2+4C$$

$$3 = 3B+C \qquad [\text{where } C = \tfrac{13}{4}]$$
$$3 = 3B+\frac{13}{4} \qquad 3B = \frac{-1}{4} \qquad B = \frac{-1}{12}$$

$$3A-6B = 0$$
$$3A = 6B$$
$$3A = 6\times\frac{-1}{12} \quad \therefore \quad A = \frac{-1}{6}$$

$$Y(s) = \frac{\frac{-1}{6} - (-\frac{1}{12})s}{s^2+4} + \frac{\frac{13}{4}}{3s-6}$$

$$= \frac{\frac{-1}{6}}{s^2+4} - \frac{\frac{1}{12}s}{s^2+4} + \frac{\frac{13}{4}}{3s-6}$$

$$= \frac{-1}{6}\cdot\frac{1}{s^2+2^2} - \frac{1}{12}\frac{s}{s^2+2^2} + \frac{13}{4}\cdot\frac{1}{3(s-2)}$$

$$= \frac{-1}{6}\cdot\frac{1}{2}\left[\frac{2}{s^2+2^2}\right] - \frac{1}{12}\left[\frac{s}{s^2+2^2}\right] + \frac{13}{12}\left[\frac{1}{s-2}\right]$$

$$y(t) = \frac{-1}{12}\sin 2t - \frac{1}{12}\cos 2t + \frac{13}{12}e^{2t}$$

$$y(t) = \frac{1}{12}\left[-\sin 2t - \cos 2t + 13e^{2t}\right]$$

$$y(t) = \frac{1}{17}\left[13e^{2t} - \cos 2t - \sin 2t \right].$$

QUESTION 3A

3

$\frac{dy}{dt} - 4y = 8 \quad t=0, y=2 \quad y(s) = 2$

$$y'(t) - 4y = 8$$

$$sy(s) - y(s) - 4y(s) = \frac{8}{s}$$

$$sy(s) - 2 - 4y(s) = \frac{8}{s}$$

$$y(s)[s-4] = \frac{8}{s} + \frac{2}{1} \cdot \frac{s+2s}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

A: $\left.\frac{8+2s}{s-4}\right|_{s=0} = \frac{8}{-4} = -2$

B: $\left.\frac{8+2s}{s}\right|_{s=4} = \frac{8+2(4)}{4} = 4$

$$y(s) = \frac{-2}{s} + \frac{4}{s-4}$$

$$y(t) = -2 + 4e^{4t}$$

Question 4

4 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2t}$ at $t=0$ $y=2$, $y'(0) = 1$

$y(0) = 2$ $y''(0) = 1$

$$y'' - 2y' + 5y = e^{2t}$$

$$(s^2y(s) - sy(0) - y'(0)) = -2(sy(s) - y(0)) + 5y(s)$$

$$= \frac{1}{s-2}$$

$$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$$

$$y(s)[s^2 - 2s + 5] = \frac{1}{s-2} + \frac{2s}{1}\frac{-3}{1} = \frac{1+2s(s-2)-3(s-2)}{s-2}$$

$Y(s)[s^2-7s+5) = \frac{1+7s^2-4s-35+6}{s-2} = \frac{2s^2-7s+7}{s-2}$

$Y(s) = \frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs+C}{s^2-2s+5}$

$A: \left.\frac{2s^2-7s+7}{s^2-2s+5}\right|_{s=2} = \frac{2(2)^2-7(2)+7}{2^2-2(2)+5} = \frac{1}{5}$

$2s^2-7s+7 = A(s^2-2s+5) + (Bs+C)(s-2)$

$2s^2-7s+7 = As^2-2As+5A + Bs^2-2Bs+Cs-2C$

$2 = A+B$

$2 = \frac{1}{5}+B \quad B = \frac{9}{5}$

$7 = 5A-2C$

$7 = 5(\frac{1}{5})-2C$

$7-1 = -2C, \quad C = -3$

$Y(s) = \frac{\frac{1}{5}}{s-2} + \frac{\frac{9}{5}s-3}{s^2-2s+5} = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{\frac{9}{5}s}{(s+1)^2+4} - \frac{3}{(s+1)^2+4}$

$Y(s) = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\frac{s+1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$

$Y(s) = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\frac{s+1}{(s+1)^2+2^2} - \frac{4}{(s+1)^2+2^2}\cdot\frac{2}{2}$

$Y(s) = \frac{1}{5}\cdot\frac{1}{(s-2)} + \frac{9}{5}\frac{s+1}{(s+1)^2+2^2} - 2\cdot\frac{2}{(s+1)^2+2^2}$

$y(t) = \frac{1}{5}e^{2t} + \frac{9}{5}e^{-t}\cos 2t - 2e^{-t}\sin 2t$

$y(t) = \frac{1}{5}\left[e^{2t} + 9e^{-t}\cos 2t - 10e^{-t}\sin 2t\right]$

$= \frac{1}{5}\left[e^{2t} + e^{-t}(9\cos 2t - 10\sin 2t)\right]$

Question 5

5. $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0$, $y=0$, $y'=2$

$$y''(t) - 6y'(t) + 8y = e^{3t}$$

$$s^2Y(s) - sy(0) - y'(0) - 6[sY(s) - y(0)] + 8Y(s) = \frac{1}{s-3}$$

$$s^2Y(s) - 2 - 6sY(s) + 8Y(s) = \frac{1}{s-3}$$

$$Y(s)[s^2 - 6s + 8] = \frac{1}{s-3} + \frac{2}{1} = \frac{1+2(s-3)}{s-3} = \frac{1+2s-6}{s-3}$$

$$Y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{2s-5}{(s-3)(s-2)(s-4)} =$$

$$\frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

A: $\left.\frac{2s-5}{(s-2)(s-4)}\right|_{s=3} = \frac{2(3)-5}{(3-2)(3-4)} = -1$

B: $\left.\frac{2s-5}{(s-3)(s-4)}\right|_{s=2} = \frac{2(2)-5}{(2-3)(2-4)} = \frac{-1}{2}$

C: $\left.\frac{2s-5}{(s-3)(s-2)}\right|_{s=4} \quad \frac{2(4)-5}{(4-3)(4-2)} = \frac{3}{2}$

$$Y(s) = \frac{-1}{s-3} - \frac{1}{2}\cdot\frac{1}{s-2} + \frac{3}{2}\cdot\frac{1}{s-4}$$

$$y(t) = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$