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MATRIC NO: - 15/ENGO1/004

DEPARTMENT - CHEMICAL ENGINEERING

1.  $\frac{dy}{dt} + 3y = e^{-2t}$  given that at  $t=0$ ,  $y=2$

$$L\left\{\frac{dy}{dt}\right\} = sy(s) - y(0)$$

$$L\{3y\} = 3y$$

$$L\{e^{-2t}\} = \frac{1}{s+2}$$

$$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$$

$$sy(s) + 3y(s) - 2 = \frac{1}{s+2}$$

$$y(s)(s+3) = \frac{1}{s+2} + 2$$

$$y(s) = \frac{1 + 2s + 4}{(s+2)(s+3)}$$

$$y(s) = \frac{2s + 5}{(s+2)(s+3)}$$

$$\frac{2s + 5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$2s + 5 = A(s+3) + B(s+2)$$

$$2s + 5 = As + 3A + Bs + 2B$$

$$A + B = 2 \quad \times 3$$

$$3A + 2B = 5 \quad \times 1$$

$$3A + 3B = 6$$

$$3A + 2B = 5$$

$$B = 1$$

From eqn 1

$$A + 1 = 2$$

$$A = 2 - 1 = 1$$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{1}{s+2} + \frac{1}{s+3}\right\}$$

$$= e^{-2t} + e^{-3t}$$

2.  $3 \frac{dy}{dt} - 6y = \sin 2t$ . Given that  $t=0, y=1$

$$L\left\{3 \frac{dy}{dt}\right\} = 3\{sy(s) - y(0)\}$$

$$L\{-6y\} = -6y(s)$$

$$L\{\sin 2t\} = \frac{2}{s^2+2^2}$$

$$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+2^2}$$

$$3sy(s) - 6y(s) - 3 = \frac{2}{(s+2)^2}$$

$$y(s)(3s-6) = \frac{2}{(s+2)^2} + 3$$

$$y(s)(3s-6) = \frac{2+3(s+2)^2}{(s+2)^2}$$

$$y(s) = \frac{2+3(s+2)^2}{(s+2)^2(3s-6)}$$

$$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{3s-6}$$

$$2 + 3(s+2)^2 = A(s+2)(3s-6) + B(3s-6) + c(s+2)^2$$

$$2 + 3s^2 + 12s + 12 = A3s^2 - A12 + 3Bs - 6B + cs^2 + 4cs + 4c$$

$$3A + c = 3 \quad \text{--- (1)}$$

$$3B + 4c = 12 \quad \text{--- (2)}$$

$$-12A - 6B + 4c = 14 \quad \text{--- (3)}$$

From

$$3A = 3 - c$$

$$A = \frac{3 - c}{3}$$

$$3B + 4c = 12$$

$$-12 \left( \frac{3 - c}{3} \right) - 6B + 4c = 14$$

$$-12 + 4c - 6B + 4c = 14 \quad \times 6$$

$$-6B + 8c = 26$$

$$-18B - 24c = -72$$

$$-18B + 24c = 78$$

$$-48c = -150$$

$$c = +\frac{25}{8}$$

From (2)

$$3B = 12 - 4\left(\frac{25}{8}\right)$$

$$B = -\frac{1}{6}$$

From (1)

$$3A = 3 - c$$

$$A = 3 - \frac{\left(\frac{25}{8}\right)}{3}$$

$$A = -\frac{1}{24}$$

$$\frac{2 + 3(s+2)^2}{(s+2)^2(3s-6)} = \frac{-\frac{1}{24}}{(s+2)} - \frac{1}{6} + \frac{\frac{25}{8}}{(3s-6)}$$

$$\mathcal{L}^{-1} \{ y(s) \} = \mathcal{L}^{-1} \left\{ \frac{-\frac{1}{24}}{(s+2)} - \frac{1}{6} + \frac{25}{8} \right\}$$

$$y = -\frac{1}{24} e^{-2t} - \frac{1}{6} t e^{-4t} + \frac{25}{8} e^{3t}$$

$$y = -\frac{1}{6} \left( \frac{e^{-2t}}{4} - t e^{-4t} + \frac{25}{4} e^{3t} \right)$$

3.  $\frac{dy}{dt} - 4y = 8$  given that  $t=0, y=2$

$$\mathcal{L} \left\{ \frac{dy}{dt} \right\} = s y(s) - y(0)$$

$$\mathcal{L} \{ -4y \} = -4y(s)$$

$$\mathcal{L} \{ 8 \} = 8/s$$

$$s y(s) - y(0) - 4y(s) = 8/s$$

$$s y(s) - 4y(s) - y(0) = 8/s$$

$$y(s) (s-4) - 2 = \frac{8}{s} + 2$$

$$y(s) (s-4) = \frac{8+2s}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)}$$

$$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$8+2s = A(s-4) + Bs$$

$$A+B = 2$$

$$-4A = 8$$

$$A = -2$$

$$B = 2 + 2$$

$$B = 4$$

$$\frac{s+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{(s-4)}$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{-2}{s} + \frac{4}{s-4}\right\}$$

$$y = -2 + 4e^{4t}$$

$$4. \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 3y = e^{2t}$$

Given that  $t=0, y=2, y'=1$

$$L\left\{\frac{d^2y}{dt^2}\right\} = s^2 y(s) - sy(0) - y'(0)$$

$$L\left\{-2\frac{dy}{dt}\right\} = -2sy(s) + 2y(0)$$

$$L\{sy\} = sy(s)$$

$$L\{e^{2t}\} = \frac{1}{s-2}$$

$$s^2 y(s) - sy(s) - y'(0) - 2sy(s) + 2y(0) + 3y(s) = \frac{1}{s-2}$$

$$s^2 y(s) - 2sy(s) + 5y(s) - 2s - 1 + 4 = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 3$$

$$y(s)(s^2 - 2s + 5) = \frac{(2s-3)(s-2)}{(s-2)}$$

$$y(s) = \frac{1 + 2s^2 - s + 6}{(s-2)(s^2 - 2s + 5)}$$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{(s-2)} + \frac{Bs + C}{(s^2 - 2s + 5)}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$$

$$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

$$A + B = 2 \quad \text{--- (1)}$$

$$-2A - 2B + C = -7 \quad \text{--- (2)}$$

$$5A - 2C = 7 \quad \text{--- (3)}$$

From (1)

$$B = 2 - A$$

From (2)

$$-2A - 2(2 - A) + C = -7$$

$$-2A - 4 + 2A + C = -7$$

$$C = -3$$

From (3)

$$3A - 2(-3) = 7$$

$$3A = 7 - 6$$

$$A = \frac{1}{3}$$

$$A + B = 2$$

$$\frac{1}{3} + B = 2$$

$$B = 2 - \frac{1}{3}$$

$$B = \frac{5}{3}$$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{\frac{1}{3}}{(s-2)} + \frac{\frac{5}{3}s - 3}{(s^2 - 2s + 5)}$$

$$= \frac{1}{3(s-2)} + \frac{5s - 9}{3(s^2 - 2s + 5)}$$

$$= \frac{1}{s+2} + \frac{9}{s} \left( \frac{s-1+1}{(s-1)^2+4} \right) - \frac{3}{2} \left( \frac{2}{(s-1)^2+4} \right)$$

$$L^{-1}\{y(s)\} = L^{-1} \left\{ \frac{1}{s-2} + \frac{9}{s} \left[ \frac{s-1}{(s-1)^2+2^2} - 1 \times \frac{2}{(s-1)^2+2^2} \right] \right.$$

$$\left. - \frac{3}{2} \left( \frac{2}{(s-1)^2+2^2} \right) \right\}$$

$$= L^{-1} \left\{ \frac{1}{s-2} + \frac{9}{s} \left[ \frac{s-1}{(s-1)^2+2^2} + \frac{1}{2} \left( \frac{2}{(s-1)^2+2^2} \right) \right] \right.$$

$$\left. - \frac{3}{2} \left( \frac{2}{(s-1)^2+2^2} \right) \right\}$$

$$y = \frac{1}{s} e^{2t} + \frac{9}{s} \left[ e^t \cos 2t + \frac{1}{2} e^t \sin 2t - \frac{3}{2} (e^t \sin 2t) \right]$$

5.  $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$  given at  $t=0, y=0, y'=2$

$$L \left\{ \frac{d^2y}{dt^2} \right\} = s^2 y(s) - sy(0) - y'(0)$$

$$L \left\{ 6\frac{dy}{dt} \right\} = -6y(s) + 6y(0)$$

$$L \{ 8y \} = 8y(s)$$

$$L \{ e^{3t} \} = \frac{1}{s-3}$$

$$s^2 y(s) - sy(0) - y'(0) - 6sy(s) + 6y(0) + 8y(s) = \frac{1}{s-3}$$

$$s^2 y(s) - 6sy(s) + 8y(s) - 2 = \frac{1}{s-3}$$

$$y(s)(s^2 - 6s + 8) = \frac{1}{s-3} + 2$$

$$y(s)(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$2s-5 = A(s^2-6s+8) + (Bs+C)(s-3)$$

$$2s-5 = As^2 - 6As + 8A + Bs^2 - 3Bs + Cs - 3C$$

$$A+B=0$$

$$-6A-3B+C=2$$

$$8A-3C=-3$$

$$B=-A \text{ from (1)}$$

$$-6A+3A+C=2$$

$$-3A+C=2 \quad \text{--- x}$$

$$8A-3C=-3 \quad \text{--- (*)}$$

$$9A-3C=-6$$

$$8A-3C=-3$$

$$A=-1$$

$$B=1$$

From (\*)

$$C=2-3$$

$$C=-1$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{s-3} + \frac{s-1}{(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{1}{s-3} + \frac{s-1}{(s-2)(s-4)}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$$s-1 = A(s-4) + B(s-2)$$

$$s-1 = As - 4A + Bs - 2B$$

$$A + B = 1 \quad \text{--- (1)}$$

$$-4A - 2B = -1 \quad \text{--- (2)}$$

$$-4A - 4B = -4$$

$$-4A - 2B = 1$$

$$-2B = -3$$

$$B = \frac{3}{2}$$

$$A = -\frac{1}{2}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{-\frac{1}{2}}{s-2} + \frac{\frac{3}{2}}{s-4}$$

$$\frac{2s-5}{(s-3)(s^2-cs+6)} = \frac{-1}{s-3} + \left( \frac{-\frac{1}{2}}{s-2} + \frac{\frac{1}{4}}{s-4} \right)$$

$$L^{-1}(y(s)) = L^{-1} \left\{ \frac{-1}{s-3} - \frac{1}{2} \frac{1}{s-2} + \frac{3}{2} \frac{1}{s-4} \right\}$$

$$y = -e^{3t} - \frac{1}{2} e^{2t} + \frac{3}{2} e^{4t}$$

$$y = \frac{1}{2} \left[ 2e^{3t} + e^{2t} - 3e^{4t} \right]$$