EKANEM EFFIONG
IS/ENG07/017
PETROLEUM ENGINEERING
ENG 381 ASSIGNMENT IV.

① $\frac{dy}{dt} + 3y = e^{-2t}$ given that at $t=0, y=2$

$L\{\frac{dy}{dt}\} = sy(s) - y(s)$

$L[3y] = 3y$

$L[e^{-2t}] = \frac{1}{s+2}$

$sy(s) - y(s) + 3y(s) = \frac{1}{s+2}$

$sy(s) + 3y(s) - 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1+2(s+2)}{(s+2)}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

$2s+5 = As + 3A + Bs + 2B$

$A+B = 2 * 3$

$3A+2B = 5 * 1$

$3A+3B = 6$ ----- ⊛

$3A+2B = 5$ ----- ⊛

$B = 1$

From equation ①

$\frac{2s+5}{(s+2)(s-3)} = \frac{1}{s+2} + \frac{1}{s+3}$

$L^{-1}[y(s)] = L^{-1}\{\frac{1}{s+2} + \frac{1}{s+3}\}$

$y = e^{-2t} + e^{-3t}$

② $\frac{3dy}{dt} - 6y = \sin 2t$ given that at $t = 0$, $y = 1$

$L\left[3dy/dt\right] = s\{dy(s) - y(s)\}$

$L[-6y] = -6y(s)$

$L[\sin 2t] = \frac{2}{s^2+2^2}$

$3y(s) - 3y(0) - 6y(s) = \frac{2}{s^2+2^2}$

$3dy - 6y(s) - 3 = \frac{2}{(s+2)^2}$

$y(s)(3s-6) = \frac{2+3(s+2)^2}{(s+2)^2}$

$y(s) = \frac{2+3(s+2)^2}{(s+2)^2(3s-6)}$

$\frac{2+3(s+2)^2}{(s+1)^2(3s-6)} = \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{(3s-6)}$

$\frac{2+3(3+2)^2}{(s+1)^2(3s-6)} = \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{(3s-6)}$

$2+3s^2+12s+12 = As^2 - A12 + 3Bs - 6B + (2s^2 +$

$3A + C = 3$ --- (1)

$3B + 4C = 12$ --- (2)

$-RA - 6B + 4C = 14$ --- (3)

From (1)

$3A = 3 - C$

$A = \frac{3-C}{3}$

$3B + 4C = 12$

$-12\left(\frac{3-C}{3}\right) - 6B + 4C = 14$

$-12 + 4C - 6B + 4C = 19$

$-6B + 8C = 28$

$-18B - 24C = -7i$

$-18B + 24C = 84$

$48C = -56$

$L = 13/6$

$8 + 25 = A(s-4) + Bs$

No1 Continuation

$3A = 3 - C$

$3A = 3 - 13/4$

$A = -1/12$

$\frac{2+3(s+2)}{(s+2)^2(s-6)} = \frac{-1/2}{(s+2)} - \frac{1/3}{(s+2)^2} + \frac{13/4}{(2s-6)}$

$L^{-1}[y(s)] = L^{-1}\left[\frac{-1/2}{(s+2)} - \frac{1/3}{(s+2)^2} + \frac{13/4}{(3s-6)}\right]$

$y = -1/12 e^{-2t} - 1/3 t e^{-1t} + 13/12 e^{3t}$

$= y = 1/12(e^{-2t} + 4te^{-4t} + 13e^{3t})$ //

No 2 Continuation

$8 + 2s = As - 4A + Bs$

$A + B = 2$

$-4A = 8$

$A = -2$

$B = 2 + 2, \; B = 4$

$\frac{8+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{(s-4)}$

$L^{-1}[y(s)] = L^{-1}\left[\frac{2}{3} + \frac{4}{s-4}\right]$

$y = -2 + 4e^{4t}$ //

(3) $\frac{dy}{dt} - 4y = 8$ given that $t=0, y=2$

$L\left[\frac{dy}{dt}\right] = sy(s) - y(t)$

$L[-4y] = -4y(s)$

$L[8] = 8/s$

$sy(s) - y(0) - 4y(s) = 8/s$

$sy(s) - 4y(s) - y(0) = 8/s$

$y(s)(s-4) = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)}$

$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{(s-4)}$

$A + B = 0$

$-6A - 3B + C = 2$

$8A - 3I = -3$

$B = -A$ from (1)

$-6A + 3A + C = 2$

$-3A + C = 2 \cdots (1) * -3$

$8A - 3C = -5 \cdots (2) * 1$

$9A - 3C = -6$

$8A - 3C = -5$

$A = -1, \; B = 1$

(4) $\frac{d^2y}{dt^2} - \frac{2dy}{dt} + 5y = e^{2t}$ given that at $y=3, y=1$

$L\left[\frac{d^2y}{dt^2}\right] = s^2y(s) - sy(s) - y'(0)$

$L\left[-2\frac{dy}{dt}\right] = -2sy(s) + 2y(0)$

$L[5y] = 5y(s)$

$L[4^{2t}] = \frac{1}{s-2}$

$s^2y(s) - sy(s) - y'(0) - 2sy(s) + 2y(0) + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2sy(s) + 5y(s) - 2s - 1 + 4 = \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 3$

$y(s)(s^2 - 2s + 5) = \frac{(2s-3)(s-2)}{(s-2)}$

$y(s) = \frac{1 + 2s^2 - 5s + 6}{(s-2)(s^2 - 2s + 5)}$

$\frac{s-1}{(s-2)(s-4)} = \frac{-1/2}{s-2} + \frac{3/2}{s-4}$

$\frac{2s-5}{(s-3)(s^2-6s+8)(s-3)} = 1 + \left(\frac{-1/2}{(s-2)} + \frac{3/2}{(s-4)}\right)$

$L^{-1}[y'(s)] = L^{-1}\left[\frac{1}{(s-3)} - \frac{1/2}{(s-2)} + \frac{3/2}{(s-4)}\right]$

$y = e^{3t} - 1/2 e^{2t} + 3/2 e^{4t}$

$y = 1/2(2e^{3t} + e^{2t} - 3e^{4t})$

$L = 2 - 3$

$L = -1$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \frac{s-1}{(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \frac{s-1}{(s-2)(s-4)}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{(s-4)}$$

$$s-1 = A(s-4) + B(s-2)$$

$$s-1 = As - 4A + Bs - 2B$$

$A + B = 1 \quad \cdots \quad ① * -4$

$-4A - 2B = -1 \quad \cdots \quad ② * 1$

$-4A - 4B = -4$

$-4A - 2B = -1$

$-2B = -3$

$B = 3/2$

$A = -1/2$ //