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16/ENG06/087 MECHANICAL ENGINEERING
ENG 307 (ASSIGNMENT)

(2). Assignment 5

i) $\frac{dy}{dt} + 3y = e^{-2t}$ at $t=0$ $y=2$

$y' + 3y = e^{-2t}$

$5y(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$y(s)[s+3] - 2 = \frac{1}{s+2}$

$(s+3)y(s) = \frac{1}{s+2} + 2$

$y(s)[s+3] = \frac{1+2(s+2)}{s+2}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)}$

$y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$2s+5 = A(s+3) + B(s+2)$

$s = -3$

$-1 = -B$

$B = 1$

$s = -2$

$1 = A$

$= \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = e^{-2t} + e^{-3t}$

ii) $3\frac{dy}{dt} - 6y = \sin 2t$ at $t=0$ $y=1$

$3y' - 6y = \sin 2t$

$3[sy(s) - y(0)] - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+4}$

$y(s)[3s-6] - 3 = \frac{2}{s^2+4}$

$y(s)[3s-6] = \frac{2}{s^2+4} + 3$

$y(s) = \frac{2+3(s^2+4)}{(s^2+4)(3s-6)}$

$= \frac{2+3s^2+12}{(s^2+4)(3s-6)}$

$= \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{As+B}{s^2+4} + \frac{C}{3s-6}$

$3s^2+14 = As+B(3s-6) + C(s^2+4)$

$s=2$

$26 = C(4+4)$

$C = \frac{26}{8} = 13/4$

Coefficient of s^2

$3 = 3B + C$

$\frac{3-C}{3} = B$

$B = \frac{3 - 13/4}{3} = -1/12$

Coefficients of s:

$0 = 3A - 6B$

$A = \frac{6B}{3} = 2B = 2(-1/12)$

$= -1/6$

$y(s) = \frac{-1/6 - 1/12 s}{s^2+4} + \frac{13}{4}\cdot\frac{1}{3s-6}$

$y(s) = -1/6 \cdot \frac{1}{s^2+4} - \frac{1}{12}s\cdot\frac{1}{s^2+4} + 13/4 \frac{1}{3(s-2)}$

$y(s) = -\frac{1}{12}\cdot\frac{2}{s^2+4} - \frac{1}{12}\cdot\frac{s}{s^2+4} + \frac{13}{12}\cdot\frac{1}{s-2}$

$y(t) = -\frac{1}{12}\sin 2t - \frac{1}{12}\cos 2t + \frac{13}{12}e^{2t}$

$y(t) = e^{2t} - \frac{1}{12}\left[\sin 2t + \cos 2t - 13e^{2t}\right]$

Ans the assignment solution.

B = 2

A = 3.

(iii) $\frac{dy}{dt} - 4y = 8 \quad t = 0 \; y = 2$

$y' - 4y = 8.$

$sy(s) - y(0) - 4y(s) = 8/s.$

$y(s)[s-4] - 2 = 8/s$

$y(s) = \frac{8}{5} + 2\cdot\frac{1}{s-4}.$

$y(s) = \frac{8+2s}{s}\cdot\frac{1}{s-4}.$

$= \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}.$

$8+2s = A(s-4) + Bs.$

$s = 4.$

$8 + 2(4) = 4B$

$8+8 = 4B.$

$4B = 16$

$B = 4.$

$s = 0.$

$8 = -4A$

$A = -2$

$y(s) = \frac{4}{s-4} - \frac{2}{s}.$

$y(t) = 4e^{4t} - 2$

(iv) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$t = 0 \quad y = 2, \; y' = 1.$

$y'' - 2y' + 5y = e^{2t}$

$s^2y(s) - sy'(0) - y(0) - 2sy(s) + 2y(0) + 5(y(s)) = \frac{1}{s-2}.$

$s^2y(s) - sy(0) - 1 - 2sy(s) + 2(2) + 5y(s) = \frac{1}{s-2}$

$y(s)[s^2 - 2s + 5] - 2s - 1 + 4 = \frac{1}{s-2}$

$y(s)[s^2 - 2s + 5] = \frac{1 - 3s + 6 + 2s^2 - 4s}{s-2}$

$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)}.$

$y(s) = \frac{A}{s-2} + \frac{Bs+C}{s^2-2s+5}.$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs+C)(s-2)$

$s = 2$

$8 - 14 + 7 = A(4 - 4 + 5)$

$1 = 5A \Rightarrow A = 1/5$

Coefficient of s^2

$2 = A + B$

$B = 2 - 1/5$

$B = 9/5.$

Of coefficient of 5.

$-7 = -2A - 2B + C$

$C = -7 + 2A + 2B$

$= -7 + \frac{2}{5} + \frac{18}{5}$

$C = -3$

$$Y(s) = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{\frac{9}{5}s - 3}{s^2 - 2s + 5}$$

$$= \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\frac{s}{s^2-2s+5} - \frac{3}{s^2-2s+5}$$

$$= \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\frac{s-1+1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$$

$$= \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\frac{s+1}{(s+1)^2+4} - \frac{1}{(s+1)^2+4} - \frac{3}{2}\frac{2}{(s+1)^2+2^2}$$

$$y(t) = \frac{1}{5}e^{2t} + \frac{9}{5}e^{-t}\cos 4t - \frac{1}{2}e^{-t}\sin 2t - \frac{3}{2}e^{-t}\sin 2t$$

$$y(t) = \frac{1}{5}e^{2t} + \frac{9}{5}e^{-t}\cos 4t - 2e^{-t}\sin 2t.$$

(v) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$

$t = 0,\ y = 0,\ y' = 2.$

$y'' - 6y' + 8y = e^{3t}.$

$s^2y(s) - sy(0) - y'(0) - 6[sy(s) - y(0)] + 8y(s)$

$= \frac{1}{s-3}.$

$s^2y(s) - 0 - 2 - 6sy(s) + 0 + 8y(s)$

$= \frac{1}{s-3}$

$y(s)[s^2 - 6s + 8] = \frac{1}{s-3} + 2.$

$y(s) = \frac{1 + 2(s-3)}{s-3}\cdot\frac{1}{s^2-6s+8}.$

$$Y(s) = \frac{1 + 2s - 6}{(s-3)(s^2-6s+8)}$$

$$Y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$Y(s) = \frac{2s-5}{(s-3)(s-4)(s-2)} = \frac{A}{(s-3)} + \frac{B}{(s-4)} + \frac{C}{(s-2)}$$

A: $\left.\frac{2s-5}{(s-4)(s-2)}\right|_{s=3} = \frac{2(3)-5}{(-1)(1)}$

$= -1$

B: $\left.\frac{2s-5}{(s-3)(s-2)}\right|_{s=4} = \frac{2(4)-5}{(4-3)(4-2)}$

$= \frac{3}{2}.$

C: $\left.\frac{2s-5}{(s-3)(s-4)}\right|_{s=2} = \frac{2(2)-5}{(-1)(-2)}.$

$= -\frac{1}{2}.$

$$Y(s) = \frac{-1}{s-3} + \frac{3}{2}\cdot\frac{1}{s-4} - \frac{1}{2}\cdot\frac{1}{s-2}$$

$= -e^{3t} + \frac{3}{2}e^{4t} - \frac{1}{2}e^{2t}.$