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Assignment 5

① $\frac{dy}{dt} + 3y = e^{-2t}$ given that at $t=0$, $y=2$

$L\{dy/dt\} = sy(s) - y(0)$

$L\{3y\} = 3y$

$L\{e^{-2t}\} = \frac{1}{s+2}$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) + 3y(s) - 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1+2(s+2)}{(s+2)}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

$2s+5 = As+3A+Bs+2B$

$A+B = 2 \times 3$

$3A+2B = 5 \times 1$

$3A+3B = 6$

$3A+2B = 5$

$B = 1$

from eqn ①

$A+1 = 2$

$A = 2-1 = 1$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{1}{(s+2)} + \frac{1}{s+3}\right\}$$

$$= e^{-2t} + e^{-3t}$$

(2) $3\frac{dy}{dt} - 6y = \sin 2t$ given that $y(0) = 1$

$$L\{3\,dy/dt\} = 3\{sy(s) - y(0)\}$$

$$L\{-6y\} = -6y(s)$$

$$L\{\sin 2t\} = \frac{2}{s^2+2^2}$$

$$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+2^2}$$

$$3sy(s) - 6y(s) - 3 = \frac{2}{(s+2)^2}$$

$$y(s)(3s-6) = \frac{2}{(s+2)^2} + 3$$

$$y(s)(3s-6) = \frac{2+3(s+2)^2}{(s+2)^2}$$

$$y(s) = \frac{2+3(s+2)^2}{(s+2)^2(3s-6)}$$

$$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{3s-6}$$

$$2+3(s+2)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$$

$$2+3s^2+12s+12 = As^2 - 12A + 3Bs + 3Bs - 6B + Cs^2 + 4Cs + 4C$$

$$3A + C = 3 \quad -(1)$$

$$3B + 4C = 12 \quad -(2)$$

$$-12A - 6B + 4C = 14 \quad -(3)$$

from (1)

$$3A = 3 - C$$

$$A = \frac{3-C}{3}$$

$$3B + 4C = 12$$

$$-12\left(\frac{3-C}{3}\right) - 6B + 4C = 14$$

$-12+4c-6B+4C=14 \times -6$

$-6B+8C=26$

$-18B-24C=-72$

$-18B+24c=78$

$-48C=-130$

$C=+\frac{25}{8}$

from (2)

$3B=12-4\left(\frac{25}{8}\right)$

$B=4/6$

from (1)

$3A=3-C$

$A=3-(25/8)$

3

$A=-1/24$

$$\frac{2s+3}{(s+2)^2(3s-6)} = \frac{-1/24}{(s+2)} - \frac{1/6}{(s+2)^2} + \frac{25/8}{(3s-6)}$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{-1/24}{(s+2)} - \frac{1/6}{(s+2)^2} + \frac{25/8}{3s-6}\right\}$$

$$y = \frac{-1}{24}e^{-2t} - \frac{1}{6}te^{-4t} + \frac{25}{24}e^{3t}$$

$$y = -\frac{1}{6}\left(\frac{1}{4}e^{-2t} - te^{-4t} + \frac{25}{4}e^{3t}\right)$$

(3) $\frac{dy}{dt} - 4y = 8$ given that $t=0$, $y=2$

$L\{dy/dt\} = sy(s) - y(0)$

$L\{-4y\} = -4y(s)$

$L\{8\} = 8/s$

$sy(s) - y(0) - 4y(s) = 8/s$

$sy(s) - 4y(s) - y(0) = 8/s$

$y(s)(s-4) - 2 = \frac{8}{s} + 2$

$y(s)(s-4) = \frac{s+2s}{s(s-4)}$

$y(s) = \frac{s+2s}{s(s-4)}$

$\frac{s+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$s+2s = A(s-4) + Bs$

$A+B = 2$

$-4A = s$

$A = -2$

$B = 2+2$

$B = 4$

$\frac{s+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{(s-4)}$

$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{2}{s} + \frac{4}{s-4}\right\}$

$y = -2+4e^{4t}$

④ $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 3y = e^{2t}$ given $t=0$ $y=2$, $y=2$

$L\{d^2y/dt^2\} = s^2y(s) - sy(s) = y'(0)$

$L\{-2dy/dt\} = -2sy(s) + 2y(0)$

$L\{3y\} = 3y(s)$

$L\{e^{2t}\} = \frac{1}{s-2}$

$s^2y(s) - 2sy(s) - y'(0) - 2sy(s) + 2y(0) + 3y(s) = \frac{1}{s-2}$

$s^2y(s) - 2sy(s) + 3y(s) - 2s - 1 + 4 = \frac{1}{s-2}$

$y(s)(s^2-2s+3) = \frac{1}{s-2} + 2s - 3$

$y(s)(s^2-2s+3) = \frac{(2s-3)(s-2)}{(s-2)}$

$y(s) = \frac{1+2s^2-5+6}{(s-2)(s^2-2s+3)}$

$$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{A}{(s-2)} + \frac{Bs+C}{(s^2-2s+5)}$$

$$2s^2-7s+7 = A(s^2-2s+5)+(Bs+C)(s-2)$$

$$2s^2-7s+7 = As^2-2As+5A+Bs^2-2Bs+Cs-2C$$

$$A+B=2 \quad -(1)$$

$$-2A-2B+C=-7 \quad -(2)$$

$$5A-2C=7 \quad -(3)$$

from (1)

$$B=2-A$$

from (2)

$$-2A-2(2-A)+C=-7$$

$$-2A-4+2A+C=-7$$

$$C=3$$

from (3)

$$5A-2(-3)=7$$

$$A=7-6$$

$$A=1/5$$

$$A+B=2$$

$$\frac{1}{5}+B=2$$

$$B=2-1/5$$

$$B=9/5$$

$$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{1/5}{(s-2)} + \frac{9/5\,s-3}{(s^2-2s+5)}$$

$$= \frac{1/5}{s+2} + \frac{9/5\,s}{(s^2-2s+5)} - \frac{3}{(s^2-2s+5)}$$

$$= \frac{1/5}{s+2} + \frac{9}{5}\left(\frac{s-1+1}{(s-1)^2+4}\right) - \frac{3}{2}\left(\frac{2}{(s-1)^2+4}\right)$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{1/5}{(s-2)} + \frac{9}{5}\left\{\frac{s-1}{(s-1)^2+2^2} + \frac{1\times 2/2}{(s-1)^2+2^2}\right) - \frac{3}{2}\left\{\frac{2}{(s-1)^2+2^2}\right)\right\}$$

$$= L^{-1}\left\{\frac{1/5}{(s-2)} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2+2^2} + \frac{1}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right] - \frac{3}{2}\left(\frac{2}{(s-1)^2+2}\right)\right\}$$

$$y = \frac{1}{5}e^{2t} + \frac{9}{5}\left[e^t\cos 2t + \frac{1}{2}e^t\sin 2t\right] - \frac{3}{2}(e^t\sin 2t)$$

(5) $\frac{d^2y}{dt^2} - \frac{6dy}{dt} + 8y = e^{3t}$ given at $t=0$, $y=0$ $y'=2$

$$L\left\{\frac{d^2y}{dt^2}\right\} = s^2y(s) - sy(s) - y'(s)$$

$$L\left\{-6\frac{dy}{dt}\right\} = -6y(s) + 6y(s)$$

$$L\{8y\} = 8y(s)$$

$$L\{e^{3t}\} = \frac{1}{s-3}$$

$$s^2y(s) - sy(s) - y'(s) - 6sy(s) + 6y(s) + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 6sy(s) + 8y(s) - 2 = \frac{1}{s-3}$$

$$y(s)(s^2 - 6s + 8) = \frac{1}{s-3} + 2$$

$$y(s)(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{s-3} + \frac{Bs+C}{(s^2-6s+8)}$$

$$2s-5 = A(s^2-6s+8) + (Bs+C)(s-3)$$

$$2s-5 = As^2 - 6As + 8A + Bs^2 - 3A + Cs - 3C$$

$$A + B = 0$$

$$-6A - 3B + C = 2$$

$$8A - 3C = -3$$

$B = -A$ from (1)

~~from (1)~~ $-6A + 3A + C = 2$

$$-3A + C = 2 \quad -(4)$$

$$8A - 3C = -5 \quad -(5)$$

$$9A - 3C = -6$$

$$8A - 3C = -5$$

$$A = -1$$

$$B = 1$$

from (4)

$C = 2 - 3$

$C = -1$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{1}{s-3} + \frac{s-1}{(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{1}{s-3} + \frac{s-1}{(s-2)(s-4)}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$s - 1 = A(s-4) + B(s-2)$

$s - 1 = As - 4A + Bs - 2B$

$A + B = 1 \quad -(1) \times 4$

$-4A - 2B = -1 \quad -(2) \times 1$

$-4A - 4B = -4$

$-4A - 2B = -1$

$-2B = -3$

$B = \frac{3}{2}$

$A = \frac{-1}{2}$

$$\frac{s-1}{(s-2)(s-4)} = \frac{-1/2}{s-2} + \frac{3/2}{s-4}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{s-3} + \left(\frac{-1/2}{(s-2)} + \frac{3/2}{s-4}\right)$$

$$L^{-1}(y_{(s)}) = L^{-1}\left\{\frac{-1}{s-3} - \frac{1/2}{s-2} + \frac{3/2}{s-4}\right\}$$

$y = -e^{3t} - 1/2 e^{2t} + 3/2 e^{4t}$

$y = \frac{1}{2}\left[2e^{3t} + e^{2t} - 3e^{4t}\right]$