

$$1. \frac{dy}{dt} + 2y = e^{-2t}$$

$$y(0) = 1$$

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$$y(s) = L^{-1} \left\{ \frac{1}{s+2} \right\} + L^{-1} \left\{ \frac{1}{s+2} \right\}$$

$$2t + 5 = A(1+t) + B(1+t)$$

$$\text{when } t = -1$$

$$-1 = -B$$

$$B = 1$$

$$\text{when } t = -2$$

$$1 = A$$

$$A = 1$$

$$y(t) = L^{-1} \left\{ \frac{1}{s+2} \right\} + L^{-1} \left\{ \frac{1}{s+2} \right\}$$

$$= e^{-2t} + e^{-2t}$$

$$2. \frac{dy}{dt} - 6y = \sin 2t$$

$$y(0) = 1$$

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$$y(s) = \frac{2}{(s^2+4)(s-6)} + \frac{3}{(s-6)}$$

$$y(s) = L^{-1}\{y(s)\} = L^{-1}\left\{\frac{s^2+14}{(s^2+4)(s-6)}\right\} = L^{-1}\left\{\frac{As+B}{s^2+4} + \frac{C}{s-6}\right\}$$

$$s^2+14 = (As+B)(s-6) + C(s^2+4)$$

$$s^2+14 \text{ when } s=0$$

$$14 = -6B + 4C \dots \textcircled{1}$$

$$\text{when } s=2$$

$$\frac{20}{8} = \frac{8C}{8}$$

$$C = \frac{15}{4}$$

$$14 = -6B + 4\left(\frac{15}{4}\right)$$

$$14 = -6B + 15$$

$$14 - 15 = -6B$$

$$1 = -6B$$

$$B = -\frac{1}{6}$$

$$\text{when } s=1$$

$$17 = (A+B)(s-6) + C(s^2+4)$$

$$17 = -3A - 3B + 5C$$

$$17 = -3A - 3\left(-\frac{1}{6}\right) + 5\left(\frac{15}{4}\right)$$

$$17 = -3A + \frac{1}{2} + \frac{65}{4}$$

$$3A = \frac{1}{2} + \frac{65}{4} - 17$$

$$3A = -\frac{1}{4}$$

$$A = -\frac{1}{12}$$

$$y(s) = L^{-1}\left\{\frac{-\frac{1}{12}s - \frac{1}{6}}{s^2+4} + \frac{\frac{15}{4}}{s-6}\right\} = L^{-1}\left\{\frac{-1}{12(s^2+2^2)}\right\} - L^{-1}\left\{\frac{1}{6(s^2+2^2)}\right\}$$

$$= \frac{1}{12} \cos 2t - \frac{1}{12} \sin 2t + \frac{15}{4} e^{6t}$$

$$y(t) = -\frac{1}{12} \cos 2t - \frac{1}{12} \sin 2t + \frac{15}{4} e^{6t}$$

$$y(t) = \frac{15}{4} e^{6t} - \frac{1}{12} \cos 2t - \frac{1}{12} \sin 2t$$

$$y(t) = \frac{1}{12} (15e^{6t} - 6 \cos 2t - \sin 2t)$$

$$2s^2 - 11s + 7 = A(s^2 - 2s + 9) + B(s - 2)$$

$$s = 2$$

$$8 - 14 + 7 = 9A$$

$$1 = 9A$$

$$A = \frac{1}{9}$$

$$2s^2 - 11s + 7 = As^2 - 2As + 9A + Bs^2 + Cs - 2C$$

$$7 = 9A - 2C$$

$$7 = 1 - 2C$$

$$-7 = -2C$$

$$+6 = -2C$$

$$C = -3$$

$$-7 = -2A - 2A + C$$

$$-7 = -\frac{2}{9} - 2B - 3$$

$$2B = -3 - \frac{2}{9} + 7$$

$$2B = \frac{16}{9}$$

$$B = \frac{8}{9}$$

$$B = \frac{9}{15}$$

$$y(t) = L^{-1} \left(\frac{1}{5(s-2)} + \frac{9}{15} \frac{1-3}{s^2-2s+9} \right)$$

$$= L^{-1} \left(\frac{1}{5} \frac{1}{s-2} + \frac{9}{15} \frac{1-3}{s^2-2s+9} - \frac{3}{s^2-2s+9} \right)$$

$$= \frac{1}{5} e^{2t} + \frac{9}{15} e^t \cos 2t - \frac{3}{2} e^t \sin 2t$$

$$= 6e^{2t} + 18e^t \cos 2t - 45e^t \sin 2t$$

$$y'' - 6y' + 8y = e^{3t}$$

$$y'' - 6y' + 8y = e^{3t}$$

$$s^2 y(s) - sy(0) - y'(0) - 6(sy(s) - y(0)) + 8y(s) = \frac{1}{s-3}$$

$$s^2 y(s) - sy(0) - y'(0) - 6sy(s) + 6y(0) + 8y(s) = \frac{1}{s-3}$$

$$\text{at } t=0, y=0, y'=2$$

$$y(s) (s^2 - 6s + 8) - y(0)(s-6) - y'(0) = \frac{1}{s-3}$$

$$y(s) (s^2 - 6s + 8) = \frac{1 + 2s - 6}{s-3}$$

$$y(s) = \frac{2s-5}{(s-2)(s^2+8)}$$

$$y(s) = \frac{2s-5}{(s-2)(s-4)(s+2)} = \frac{A}{s-2} + \frac{B}{s-4} + \frac{C}{s+2}$$

$$2s-5 = A(s-4)(s+2) + B(s-2)(s+2) + C(s-2)(s-4)$$

when $s=2$

$$1 = -B$$

$$B = -1$$

when $s=4$

$$3 = 2C$$

$$C = \frac{3}{2}$$

when $s=0$

$$-1 = 2A$$

$$A = -\frac{1}{2}$$

$$y(s) = L^{-1} \left(-\frac{1}{2} \frac{1}{s-2} - \frac{1}{s-4} + \frac{3}{2} \frac{1}{s+2} \right)$$

$$= -\frac{1}{2} e^{2t} - e^{4t} + \frac{3}{2} e^{-2t}$$

$$= 3e^{-2t} - e^{4t} - 2e^{2t}$$