1) 15/ENG02/006

$$\frac{dy}{dt} + 3y = e^{-2t} \qquad t=0$$

$$y=2, \; y(0)=2$$

$$\left(sy(s) - y(0)\right) + 3y(s) = \frac{1}{s+2}$$

$$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$$

$$y(s)(s+3) = \frac{1+2(s+2)}{s+2} = \frac{2s+5}{s+2}$$

$$Y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

when $s = -2$ $\qquad\qquad$ $s = -3$

$$\frac{2(-2)+5}{-2+3} = A \qquad \frac{2(-3)+5}{-3+2} = B$$

$$A = \frac{-4+5}{1} = 1 \qquad B = \frac{-6+5}{1} \quad \frac{1}{2}$$

$$y(t) = L^{-1}\left[\frac{1}{s+2} + \frac{1}{2(s+3)}\right] = e^{-2t} + \frac{1}{2}e^{-3t}$$

2) $3\frac{dy}{dt} - 6y = \sin 2t$ $t=0$
$y=1$ $y(0)=1$

$3(sy(s) - y(0)) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3 - 6y(s) = \frac{2}{s^2+4}$

$y(s)(3s-6) = \frac{2 + 3(s^2+4)}{s^2+4}$

$= 2 + 3s^2 + 12 = \frac{3s^2+14}{s^2+4}$

$y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{As+B}{s^2+4} + \frac{C}{3s-6}$

$3s^2+4 = (As+B)(3s-6) + C(s^2+4)$

~~when~~ $s =$

$3s^2+4 = 3As^2 - 6A + 3Bs - 6B + Cs^2 + 4$

$3 = 3A + C$

$4 = -6A + 4$ $\qquad A = \frac{4-4}{-6}$

$0 = 3B$ $\qquad = 0$

$B = 0$

$C = 3$

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$$y(t) = L^{-1}\left[\frac{3}{3s-6}\right]$$

$$= L^{-1}\left[\frac{1}{s-2}\right] = e^{2t}$$

iii) $\frac{dy}{dt} - 4y = 8 \quad t=0$

$y=2 \quad y(0)=2$

$$(sy(s) - y(0)) - 4y(s) = \frac{8}{s}$$

$$sy(s) - 2 - 4y(s) = \frac{8}{s}$$

$$y(s)(s-4) = \frac{8+2s}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$s=0 \qquad\qquad s=4$

$$\frac{8}{-4} = A \qquad \frac{8+2(4)}{\cancel{4(4-4)}} = B = \frac{16}{4}$$

$A = -2 \qquad B = 4$

(c) $\frac{d^2y}{dt^2} - \frac{2dy}{dt} + 5y = e^{2t}$ $\quad t=0$

$y(0)=2$

$y'(0)=1$

$(s^2y(s) - sy(0) - y'(0)) - 2(sy(s) - y(0))$
$+ 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s - 1 - 2sy(s) - 4 + 5y(s) = \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) - 2s - 5 = \frac{1}{s-2}$

$y(s)(s^2-2s+5) = \frac{1 + 2s(s-2) + 5(s-2)}{s-2}$

$= \frac{1 + 2s^2 - 4s + 5s - 10}{s-2}$

$= \frac{2s^2 + s - 9}{s-2}$

$y(s) = \frac{2s^2+s-9}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{B}{s^2-2s+5}$

when $s = 2$

$A = \frac{2(2)^2 + 2 - 9}{2^2 - 2(2) + 5} = \frac{8-7}{4-4+5} = \frac{1}{5}$

$2s^2+s-9 = As^2-2s+5$

$A=2$

$$y(t) = L^{-1}\left[\frac{2}{s-2} + \frac{1}{5(s^2-2s+5)}\right]$$

$$= L^{-1}\left[\frac{2}{s-2} + \left(\frac{2}{(s-1)^2+2^2}\right)\frac{1}{10}\right]$$

$$= 2e^{2t} \frac{2}{2} + \frac{1}{10}\left(e^t \sin 2t\right)$$

v) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ $t=0$

$y(0)=0$

$y'(0)=2$

$$\left(s^2y(s) - sy(0) - y'(0)\right) - 6\left(sy(s) - y(0)\right) + 8\left(y(s)\right)$$

$$= \frac{1}{s-3}$$

$s^2 y(s) - s y(0) - y'(0) - 6s y(s) - 6y(0) + 8y(s) = \frac{1}{s-3}$

$s^2 y(s) - 2 - 6s y(s) + 8y(s) = \frac{1}{s-3}$

$y(s)(s^2 - 6s + 8) - 2 = \frac{1 + 2s - 6}{s-3}$

$= \frac{2s-5}{s-3}$

$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{s-3} + \frac{B}{s^2-6s+8}$

$s = 3$

$\frac{2(3)-5}{3^2-6(3)+8} = A$

$A = \frac{1}{9-18+8} = \frac{1}{9-10} = -1$

$2s - 5 = As^2 - 6As + 8A + Bs - 3B$

$2 = -6A + B$

$B = 2 + 6A = 2 - 6 = -4$

$$y(t) = L^{-1}\left[\frac{-1}{s-3} + \frac{-1}{s^2-6s+9}\right]$$

$$= L^{-1}\left[\frac{-1}{s-3} - \frac{1}{(s-3)^2-1}\right]$$

$$= \cancel{-e^{3t}}$$

$$= L^{-1}\left[\frac{-1}{s-3} - \frac{1}{(s-3)^2-1^2}\right]$$

$$= -e^{3t} - 1\cosh t\, e^{3t}$$