ASSIGNMENT 5 [Using Laplace

1. $\frac{dy}{dt} + 3y = e^{-2t}$ given at $t=0$ $y=2$ $y(0)=2$

$y' + 3y = e^{-2t}$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$y(s)[s+3] - y(0) = \frac{1}{s+2}$

$y(s)[s+3] - 2 = \frac{1}{s+2}$

~~$y(s) = \frac{1}{(s+2)(}$~~

$y(s)[s+3] = \frac{1}{s+2} + \frac{2}{1}$

$y(s)[s+3] = \frac{1 + 2[s+2]}{s+2}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)} = \frac{2s+5}{(s+2)(s+3)}$

$y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$A = \left.\frac{2s+5}{s+3}\right|_{s=-2} = \frac{-4+5}{-2+3} = \frac{1}{1} = 1$

$B = \left.\frac{2s+5}{s+2}\right|_{s=-3} = \frac{-6+5}{-3+2} = \frac{-1}{-1} = 1$

$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = e^{-2t} + e^{-3t}$

2. $3\frac{dy}{dt} - 6y = \sin 2t$ at $t=0$ $y=1$ $y(0)=1$

$3[Sy(s) - y(0)] - 6y(s) = \frac{s}{s^2+4}$

$3Sy(s) - 3y(0) - 6y(s) = \frac{s}{s^2+4}$

$y(s)[3s-6] - 3y(0) = \frac{s}{s^2+4}$

$y(s)[3s-6] - 3 = \frac{s}{s^2+4}$

$y(s)[3s-6] = \frac{s}{s^2+4} + \frac{3}{1}$

$y(s)[3s-6] = \frac{s+3[s^2+4]}{s^2+4}$

$y(s)[3s-6] = \frac{s+3s^2+12}{s^2+4}$

$y(s) = \frac{s+3s^2+12}{(s^2+4)(3s-6)} = \frac{s+3s^2+12}{(3s-6)(s+2)^2} = \frac{s+3s^2+12}{(3s-6)(s+2)^2}$

$= \frac{A+Bs}{s^2+4} + \frac{C}{3s-6}$

$C = \frac{s+3s^2+12}{s^2+4}\Big|_{s=2} = \frac{2+3(4)+12}{4+4} = \frac{26}{8} = \frac{13}{4}$

$3s^2+14 = (A+Bs)[3s-6] + C[s^2+4]$

$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C$

Taking only coefficients of s^2

$3 = 3B + C$

$3B = \frac{3}{1} - \frac{13}{4} = \frac{12-13}{4} = \frac{-1}{4}$

$B = \frac{-1}{12}$

Taking only coefficient of s

$$0 = 3A - 6B$$

$$B = \frac{-1}{12}$$

$$3A = 6\left(-\tfrac{1}{12}\right)$$

$$A = \frac{-1}{6}$$

$$y(s) = \frac{-\frac{1}{6} + -\frac{1}{12}s}{(s^2+4)} + \frac{13/4}{3s-6}$$

$$\cancel{y(s) = \frac{\quad}{(s^2+4)(3s-6)}}$$

$$y(s) = \frac{-1}{6(s^2+4)} - \frac{s}{12(s^2+4)} + \frac{13}{4(3s-6)}$$

$$y(s) = \frac{-1}{6}\cdot\frac{1}{2}\left[\frac{2}{s^2+4}\right] - \frac{1}{12}\left[\frac{s}{s^2+4}\right] + \frac{13}{12}\left[\frac{1}{s-2}\right]$$

$$y(t) = \frac{-1}{12}\sin 2t - \frac{1}{12}\cos 2t + \frac{13}{12}e^{2t}$$

$$y(t) = \frac{1}{12}\left[-\sin 2t - \cos 2t + 13e^{2t}\right]$$

$$y(t) = \frac{1}{12}\left[13e^{2t} - \sin 2t - \cos 2t\right]$$

3. $\frac{dy}{dt} - 4y = 8$ $\qquad t=0 \quad y=2 \quad y(0)=2$

$y' - 4y = 8$

$sy(s) - y(0) - 4y(s) = \frac{8}{s}$

$y(s)[s-4] - 2 = \frac{8}{s}$

$y(s)[s-4] = \frac{8}{s} + \frac{2}{1} = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$A = \left.\frac{8+2s}{s-4}\right|_{s=0} = \frac{8}{-4} = -2$

$B = \left.\frac{8+2s}{s}\right|_{s=4} = \frac{8+8}{4} = \frac{16}{4} = 4$

$y(s) = \frac{-2}{s} + \frac{4}{s-4}$

$y(t) = -2u(t) + 4e^{4t}$

4. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2t}$ at $t=0$ $y=2$ & $y'(0) = 1$ $y(0)=2$

$y'' - 2y' + 5y = e^{2t}$

$S^2y(s) - Sy(0) - y'(0) - 2[Sy(s) - y(0)] + 5y(s) = \frac{1}{s-2}$

$S^2y(s) - Sy(0) - y'(0) - 2Sy(s) + 2y(0) + 5y(s) = \frac{1}{s-2}$

$y(s)[s^2 - 2s + 5] = \frac{1}{s-2} + Sy(0) + y'(0) - 2y(0)$

$y(s)[s^2 - 2s + 5] = \frac{1}{s-2} + \cancel{Sy'(0) + 1y(0)\ 2y(0) - 2y(0)}$

$y(s)[s^2 - 2s + 5] = \frac{1}{s-2} + \cancel{2s - s - 2}\ 2s - 4$

$y(s)[s^2 - 2s + 5] = \frac{1}{s-2} + 2s + 1 - 4$

$y(s)[s^2 - 2s + 5] = \frac{1}{s-2} + 2s - 3$

$y(s)[s^2 - 2s + 5] = \frac{1 + 2s[s-2] - 3[s-2]}{s-2}$

$y(s)[s^2 - 2s + 5] = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$

$y(s)[s^2 - 2s + 5] = \frac{2s^2 - 7s + 7}{s-2}$

$y(s) = \frac{2s^2 - 7s + 7}{(s-2)[s^2 - 2s + 5]}$

$2s^2 - 7s + 7 = \frac{A}{s-2} + \frac{B}{s^2 - 2s + 3}$

$2s^2 - 7s + 7 = A[s^2 - 2s + 5] + B[s-2]$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs - 2B$

$-2A + B = -7$ $\quad A = 2$

$B = -7 + 4 = -3$

Scanned by CamScanner

$$y(s) = \frac{2}{s-2} - \frac{3}{s^2-2s+5}$$

$$= 2e^{2t}$$

5. $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0$, $y=0$, $y'=2$

$$y'' - 6y' + 8y = e^{3t}$$

$$s^2y(s) - sy(0) - y'(0) - 6[sy(s) - y(0)] + 8y(s) = \frac{1}{s-3}$$

$$y(s)[s^2 - 6s + 8] - sy(0) - y'(0) + 6y(0) = \frac{1}{s-3}$$

$$y(s)[s^2-6s+8] - 0 - 2 + 0 = \frac{1}{s-3}$$

$$y(s)[s^2-6s+8] = \frac{1}{s-3} + 2 = \frac{1+2s-6}{s-3} = \frac{2s-5}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)[s^2-6s+8]} = \frac{2s-5}{(s-3)(s-2)(s-4)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$A = \left.\frac{2s-5}{s^2-6s+8}\right|_{s=3} = \frac{6-5}{9-18+8} = \frac{1}{-1} = -1$$

$$B = \left.\frac{2s-5}{(s-3)(s-4)}\right|_{s=2} = \frac{4-5}{(-1)(-2)} = \frac{-1}{2} = -\frac{1}{2}$$

$$C = \left.\frac{2s-5}{(s-3)(s-2)}\right|_{s=4} = \frac{8-5}{(1)(2)} = \frac{3}{2}$$

$$y(s) = \frac{-1}{s-3} - \frac{1}{2}\cdot\frac{1}{s-2} + \frac{3}{2}\cdot\frac{1}{s-4}$$

$$= -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$

$$= 3/2\, e^{4t} - 1/2\, e^{2t} - e^{3t}$$