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ENG381 - Engineering Mathematics

① $\frac{dy}{dt} + 3y = e^{-2t}$; Given that at $t = 0$, $y = 2$

Solu

$$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$$
$$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$$
$$y(s)(s+3) = 2 + \frac{1}{s+2}$$
$$y(s) = \frac{2s+5}{(s+3)(s+2)}$$

$$y(s) = \frac{2s+5}{(s+3)(s+2)} \equiv \frac{A}{s+3} + \frac{B}{s+2}$$

$$\therefore 2s+5 = A(s+2) + B(s+3)$$

Put $s = -3$

$$2(-3)+5 = A(-3+2) + 0$$
$$-A = -1$$
$$A = 1$$

Putting $s = -2$

$$2(-2)+5 = A(-2+2) + B(-2+3)$$
$$B = 1$$

$$\therefore y(s) = \frac{1}{s+3} + \frac{1}{s+2}$$

$$y(t) = L^{-1}[y(s)] = L^{-1}\left[\frac{1}{s+3}\right] + L^{-1}\left[\frac{1}{s+2}\right]$$

$$y(t) = e^{-2t} - e^{-3t}$$

② $3\frac{dy}{dt} - 6y = \sin 2t$; Given that at $t = 0$, $y = 1$

Solu

$$3[sy(s) - y(0)] - 6y(s) = \frac{2}{s^2+4}$$
$$3sy(s) - 3 - 6y(s) = \frac{2}{s^2+4}$$
$$y(s)[3s-6] = 3 + \frac{2}{s^2+4}$$

$$y(s)[3s-6] = \frac{3s^2+14}{s^2+4}$$
$$y(s) = \frac{3s^2+14}{(3s-6)(s^2+4)}$$

$$y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)} \equiv$$

$$Y(s) = \frac{3s+14}{(s^2+4)(s-2)(3)} \equiv \frac{As+B}{s^2+4} + \frac{C}{(s-2)} + \frac{D}{3}$$

$$\frac{3s+14}{(s^2+4)(s-2)(3)} = \frac{(As+B)(3s-6) + C(3s^2+12) + D(s^2+4)(s-2)}{(s^2+4)(s-2)(3)}$$

$$\therefore 3s+14 = (As+B)(3s-6) + C(3s^2+12) + D(s^2+4)(s-2)$$

Puts $s = 2$

$$3(2)+14 = (2A+B)(6-6) + C(24) + 0$$

$$20 = 24C$$

$$C = \frac{5}{6}$$

Puts $s = 0$

$$14 = -6B + 12C - 8D$$

$$14 = -6B + 10 - 8D$$

$$4 = -6B - 8D \quad \ldots *$$

Puts $s =$

$$Y(s) = \frac{3s+14}{(s^2+4)(3s+6)} \equiv \frac{A+A}{s^2+4} + \frac{B}{3s+6}$$

$$Y(s) = 3s+14 = (B+A)(3s+6) + C(s^2+4)$$

Puts $s = -2$

$$3(-2)+14 = 0 + 8B$$

$$8 = 8B$$

$$B = 1$$

Puts $s = 1$

$$3+14 = A(3+6) + 5C$$

$$17 = 9A + 5C$$

$C = 1$; $17 = 9A + 5$

$$9A = 12$$

$$A = 4/3$$

$$Y(s) = \frac{4}{3(s^2+4)} + \frac{1}{3s+6}$$

$$Y(s) = \frac{4}{3}\cdot\frac{1}{(s^2+4)} + \frac{1}{3}\left(\frac{1}{s+2}\right)$$

$$y(t) = \mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}\left\{\frac{2}{3}\left(\frac{2}{s^2+4}\right)\right\} + \mathcal{L}^{-1}\left\{\frac{1}{3}\left(\frac{1}{s+2}\right)\right\}$$

$$y(t) = \frac{2}{3}\sin 2t + \frac{1}{3}e^{-2t}$$

(3) $\frac{dy}{dt} - 4y = 8$; Given that at $t=0$, $y=2$.

Solu

$sy(s) - y(0) - 4y(s) = \frac{8}{s}$

$sy(s) - 2 - 4y(s) = \frac{8}{s}$

$y(s)[s-4] = 2 + \frac{8}{s}$

$y(s)[s-4] = \frac{2s+8}{s}$

$y(s) = \frac{2s+8}{(s-4)(s)} \equiv \frac{A}{s-4} + \frac{B}{s}$

$2s+8 = As + B(s-4)$

Put $s=0$

$8 = -4B$

$B = -2$

Put $s = 4$

$2(4)+8 = 4A + 0$

$4A = 16$

$A = 4$

$\therefore y(s) = \frac{4}{s-4} + \frac{B}{s}$

~~$y(s) = 0$~~

$y(t) = L^{-1}\{y(s)\} = L^{-1}\left\{\frac{4}{s-4}\right\} + L^{-1}\left\{\frac{-2}{s}\right\}$

$y(t) = 4e^{4t} - 2$

(4) $\frac{d^2y}{dt^2} - \frac{2dy}{dt} + 5y = e^{2t}$; Given that at $t=0$, $y=2$, $y'=1$

Solu

$s^2y(s) - sy(0) - y'(0) - 2sy(s) + y(0) + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s - 1 - 2sy(s) + 2 + 5y(s) = \frac{1}{s-2}$

$y(s)[s^2 - 2s + 5) = \frac{1}{s-2} + 2s + 1 - 2$

$y(s)[s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 1$

$y(s)[s^2-2s+5] = \frac{1 + 2s(s-2) - 1(s-2)}{s-2}$

$y(s)(s^2-2s+5) = \frac{1 + 2s^2 - 4s - s + 2}{s-2}$

$y(s)[s^2-2s+5) = \frac{2s^2 - 5s + 3}{s-2}$

$$y(s) = \frac{2s^2 - 5s + 3}{(s-2)(s^2-2s+5)} \equiv \frac{A}{s-2} + \frac{B}{s^2-2s+5}$$

$$2s^2 - 5s + 3 = A(s^2-2s+5) + B(s-2)$$

Put $s = 2$

$$1 = 5A$$

$$A = \frac{1}{5}$$

Put $s = 1$

$$0 = 4A - B$$

$$B = 4A$$

$$B = \frac{4}{5}$$

$$y(s) = \frac{1}{5(s-2)} + \frac{4}{5(s^2-2s+5)}$$

$$y(t) = \mathcal{L}^{-1}\{y(s)\}$$

$$y(t) = \frac{1}{5}e^{2t} + \frac{2}{5}t \sin t$$

(5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ Given that at $t = 0$, $y = 0$, $y' = 2$

Solu

$$s^2y(s) - sy(0) - y'(0) - 6(sy(s) - y(0)) + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 0 - 2 - 6sy(s) + 0 + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 6sy(s) + 8y(s) = \frac{1}{s-3} + 2$$

$$y(s)[s^2 - 6s + 8] = \frac{2s-5}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$y(s) = \frac{2s-5}{(s-2)(s-3)(s-4)} \equiv \frac{A}{(s-2)} + \frac{B}{(s-3)} + \frac{C}{(s-4)}$$

$$y(s) = 2s - 5 = A(s-3)(s-4) + B(s-2)(s-4) + C(s-2)(s-3)$$

Put $s = 3$;

$$2(3) - 5 = B(1)(-1)$$

$$1 = -B$$

$$B = -1$$

Put $s = 4$;

$$2(4) - 5 = C(2)(1)$$

$$3 = 2C$$

$$C = \frac{3}{2}$$

Put $s = 2$; $2(2) - 5 = A(-1)(-2)$

$$-1 = 2A$$

$$A = \frac{-1}{2}$$

$$y(s) = \frac{-1}{2(s-2)} + \frac{-1}{(s-3)} + \frac{3}{2(s-4)}$$

$$y(t) = \mathcal{L}^{-1}\{y(s)\}$$

$$y(t) = \frac{-1}{2}e^{2t} - e^{3t} + \frac{3}{2}e^{4t}$$