15/ENG04/025

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ASSIGNMENT 5

(1) $\frac{dy}{dt} + 3y = e^{-2t}$

$$sy(s) - y(s) + 3y(s) = \frac{1}{s+2}$$

$$y(s)[s+3] = \frac{1}{s+2} + 2$$

$$y(s)[s+3] = \frac{1+2s+4}{s+2}$$

$$y(s) = \frac{1+2s+4}{(s+2)(s+3)}$$

$$A: \left.\frac{1+2s+4}{s+3}\right|_{s=-2} = \frac{1+(2)(-2)+4}{-2+3} = 1$$

$$B: \left.\frac{1+2s+4}{s+2}\right|_{s=-3} = \frac{1+2(-3)+4}{-3+2} = 1$$

$$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$$

$$y(t) = e^{-2t} + e^{-3t}$$

(2) $3\frac{dy}{dt} - 6y = \sin 2t$

$$3[sy(s) - y(s)] - 6y(s) = \frac{2}{s^2+4}$$

$$3sy(s) - 3y(s) - 6y(s) = \frac{2}{s^2+4}$$

$$3y(s)[3s-6] = \frac{2}{s^2+4} + 3 = \frac{2+3(s^2+4)}{s^2+4}$$

$$y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{(s^2+4)} + \frac{C}{3s-6}$$

$$C = \left.\frac{3s^2+14}{s^2+4}\right|_{s=2} = \frac{3(2)^2+14}{2^2+4} = \frac{13}{4}$$

$$3s^2+14 = (A+Bs)(3s-6) + C(s^2+4)$$

$$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C$$

$$3 = 3B + C$$

$$3B = -\frac{1}{4}$$

$$B = -\frac{1}{12}$$

$$3A - 6B = 0$$

$$3A = 6B$$

$$3A = 6 \times \frac{-1}{12}$$

$$A = \frac{-1}{6}$$

$$y(s) = \frac{-1}{6}\left(\frac{1}{s^2+4}\right) - \frac{1}{12}\left(\frac{s}{s^2+4}\right) + \frac{13}{4}\left(\frac{1}{3s-6}\right)$$

$$= -\frac{1}{6}\left(\frac{1}{s^2+2^2}\right) - \frac{1}{12}\left(\frac{s}{s^2+2^2}\right) + \frac{13}{12}\left(\frac{1}{s-2}\right)$$

$$y(t) = \frac{-1}{12}\sin 2t - \frac{1}{12}\cos 2t + \frac{13}{12}e^{2t}$$

B) $\frac{dy}{dt} - 4y = 8$

$$sy(s) - y(0) - 4y(s) = \frac{8}{s}$$

$$y(s)[s-4] = \frac{8+2s}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$A: \left.\frac{8+2s}{s(s-4)}\right|_{s=0} = \frac{A}{s} + \frac{B}{s-4} \qquad \frac{8}{-4} = -2$$

$$B: \left.\frac{8+2s}{s}\right|_{s=4} = \frac{8+2(4)}{4} = 4$$

$$y(s) = \frac{-2}{s} + \frac{4}{s-4}$$

$$y(t) = -2 + 4e^{4t}$$

(4) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2t}$

$$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$$

$$y(s)[s^2 - 2s + 5] = \frac{1}{s-2} + 2s - 3$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)}$$

$$A: \left.\frac{2s^2-7s+7}{s^2-2s+5}\right|_{s=2} = \frac{2(2)^2-7(2)+7}{2^2-2(2)+5} = \frac{1}{5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$$

$$= As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

comparing coefficients.

$$2 = A + B$$

$$2 = \frac{1}{5} + B$$

$$B = \frac{9}{5}$$

$$7 = 5A - 2C$$

$$7 = 5\left(\frac{1}{5}\right) - 2C$$

$$C = -3$$

$$y(s) = \frac{1}{s}\cdot\frac{1}{s-2} + \left(\frac{9s}{5} - 3\right)\frac{1}{s^2-2s+5}$$

$$y(s) = \frac{1}{s}\cdot\frac{1}{s-2} + \frac{9}{5}\,\frac{s+1}{(s+1)^2+2^2} - \frac{2}{2}\,\frac{4}{(s+1)^2+2^2}$$

$$y(s) = \frac{1}{s}\cdot\frac{1}{s-2} + \frac{9}{5}\,\frac{s+1}{(s+1)^2+2^2} - 2\cdot\frac{2}{(s+1)^2+2^2}$$

$$y(t) = \frac{1}{5}e^{2t} + \frac{9}{5}e^{-t}\cos 2t - 2e^{-t}\sin 2t.$$

6) $\dfrac{d^2y}{dt^2} - 6\dfrac{dy}{dt} + 8y = e^{3t}.$

$$s^2y(s) - sy(0) - y'(0) - 6sy(s) + 6y(0) + 8y(s) = \frac{1}{s-3}$$

$$y(s)[s^2 - 6s + 8] = \frac{1 + 2(s-3)}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)}$$

$$y(s) = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$A: \left.\frac{2s-5}{(s-2)(s-4)}\right|_{s=3} = \frac{2(3)-5}{(3-2)(3-4)} = -1$$

$$B: \left.\frac{2s-5}{(s-3)(s-4)}\right|_{s=2} = \frac{2(2)-5}{(2-3)(2-4)} = \frac{-1}{2}$$

$$C: \left.\frac{2s-5}{(s-3)(s-2)}\right|_{s=4} = \frac{2(4)-5}{(4-3)(4-2)} = \frac{3}{2}$$

$$y(t) = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$