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15/ENG07/008

CHEMICAL ENGINEERING

ENGINEERING MATHEMATICS 3

ASSIGNMENT 5

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CHEMICAL ENGINEERING
ENG 381:- ENGINEERING MATHEMATICS III

ASSIGNMENT V

1) Solve the following by laplace transform

i) $\frac{dy}{dt} + 3y = e^{-2t}$, given that at $t=0$, $y=2$.

Sol

$y' + 3y = e^{-2t}$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) + 3y(s) = \frac{1}{s+2} + 2$

$y(s) = \frac{1}{s+2} \div (s+3) + \frac{2}{(s+3)}$

$y(s) = \frac{1}{(s+2)(s+3)} + \frac{2}{(s+3)}$

Resolving $\frac{1}{(s+2)(s+3)}$ into partial fract

$\frac{1}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$

$1 = A(s+3) + B(s+2)$, let $s = -2$

$1 = \cancel{As} + A$

hence $A = 1$

let $s = -3$

$1 = A(-3+3) + B(-1)$

$B = -1$

hence $y(s) = \frac{1}{s+2} - \frac{1}{s+3} + \frac{2}{s+3}$

$y(t) = L^{-1}y(s) = e^{-2t} - e^{-3t} + 2e^{-3t}$

$y(t) = e^{-2t} + e^{-3t}$

(i) $3\frac{dy}{dt} - 6y = \sin 2t$ given that at $t=0$, $y=1$

$$3(s\,y(s) - y(0)) - 6y(s) = \frac{2}{s^2+4}$$

$$3s\,y(s) - 3 - 6y(s) = \frac{2}{s^2+4}$$

$$y(s)(3s-6) = \frac{2}{s^2+4} + 3$$

$$y(s) = \frac{2}{(s^2+4)(3s-6)} + \frac{3}{(3s-6)}$$

$$y(s) = \frac{2}{(3s-6)(s^2+4)} + \frac{1}{3(s-2)}$$

Resolving $\frac{2}{(3s-6)(s^2+4)}$ into partial fraction

$$\frac{2}{(3s-6)(s^2+4)} = \frac{A}{3s-6} + \frac{Bs+C}{s^2+4}$$

$$2 = A(s^2+4) + (Bs+C)(3s-6)$$

$$2 = As^2 + 4A + 3Bs^2 - 6Bs + 3Cs - 6C$$

$$2 = (A+3B)s^2 + 4A - 6C + s(3C-6B)$$

$$A + 3B = 0, \quad 2A - 3C = 1, \quad 3C - 6B = 0$$

if $A = -3B$,

$$\begin{array}{r} -6B - 3C = 1 \\ + \quad -6B + 3C = 0 \\ \hline -12B = 1 \end{array}$$

$$B = -\frac{1}{12}$$

$$A = -3 \times -\frac{1}{12} = \frac{1}{4}$$

$$-3C = 1 - 2 \times \frac{1}{4}$$

$$-3C = -\frac{2}{4}$$

$$C = -\frac{2}{4} \times \frac{1}{-3}$$

$$C = \frac{1}{6}$$

$$y(s) = \frac{\frac{1}{4}}{3s-6} + \frac{-\frac{1}{12}s + \frac{1}{6}}{s^2+4} + \frac{1}{3(s-2)}$$

$$y(s) = \frac{1}{12} \cdot \frac{1}{s-2} + \left(-\frac{1}{12}\right)\frac{(s+2)}{s^2+4} + \frac{1}{3(s-2)}$$

3)

$$y(s) = \frac{1}{12} \cdot \frac{1}{s-2} - \frac{1}{12}\left(\frac{s}{s^2+4}\right) - \frac{1}{12}\left(\frac{2}{s^2+4}\right) + \frac{1}{3(s-2)}$$

$$y(t) = L^{-1}\, y(s)$$

$$y(t) = \frac{1}{12}e^{2t} - \frac{1}{12}\cos 2t - \frac{1}{12}\sin 2t + \frac{1}{3}e^{2t}$$

$$y(t) = \frac{13}{12}e^{2t} - \frac{1}{12}\cos 2t - \frac{1}{12}\sin 2t$$

3) $\frac{dy}{dt} - 4y = 8$, given that $t=0$, $y=2$

$$y' - 4y = 8$$

$$sy(s) - y(0) - 4(y(s)) = \frac{8}{s}$$

$$y(s)(s-4) = \frac{8}{s} + 2$$

$$y(s) = \frac{8}{s(s-4)} + \frac{2}{s-4}$$

$$y(s) = \frac{8+2s}{s(s-4)}$$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$2s+8 = A(s-4) + B(s)$, when $s=4$.

$16 = 4B$

$B = 4$.

$As + Bs = 2s$

$A + B = 2$

$A = -2$

$$y(s) = \frac{-2}{s} + \frac{4}{s-4}$$

$$y(t) = -2 + 4e^{4t}$$

7 4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ given at $t=0,\ y=2,\ y'=1$

$$y'' - 2y' + 5y = e^{2t}$$

$$s^2y(s) - sy(0) - y'(0) - 2(sy(s) - y(0)) + 5(y(s)) = \frac{1}{s-2}$$

$$s^2y(s) - s(2) - 1 - 2(sy(s) - 2) + 5y(s) = \frac{1}{s-2}$$

$$s^2y(s) - 2s - 2sy(s) + 4 - 1 + 5y(s) = \frac{1}{s-2}$$

$$s^2y(s) - 2sy(s) + 5y(s) = \frac{1}{s-2} - \frac{3}{1} + \frac{2s}{1}$$

$$y(s)(s^2 - 2s + 5) = \frac{1 - (3s-6) + 2s(s-2)}{(s-2)}$$

$$y(s) = \frac{-3s + 7 + 2s^2 - 4s}{(s-2)(s^2-2s+5)}$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs+C}{s^2-2s+5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s - 2)$$

$$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

$$2s^2 - 7s + 7 = (A+B)s^2 + s(-2A - 2B) + 5A - 2C$$

$A + B = 2$, $\cancel{-2A - 2B = -7}$ $5A - 2C = 7$

$B = \cancel{2-A}$, $\cancel{A + B = 2}$

$\cancel{-2A - 2B = -7}$

Let $s = 2$.

$8 - 7 = 5A$

$A = \frac{1}{5}$

Compare Co-efficient of s^2

$A + B = 2$

$B = 2 - \frac{1}{5} = \frac{9}{5}$

Compare Co-efficient of Constant.

$-2C + 5A = 7$

$-2C = 7 - 1$

$C = -3$

$$y(s) = \frac{\frac{1}{5}}{s-2} + \frac{\frac{9}{5}s - 3}{s^2 - 2s + 5}$$

$$y(s) = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{\frac{9}{5}s}{(s-1)^2+4} - \frac{3}{(s-1)}$$

$$y(s) = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\cdot\frac{s}{(s-1)^2+4} - \frac{3}{(s-1)}$$

$$y(t) = \frac{1}{5}e^{2t} + \left(-\frac{3}{5}\right)e^{t}\cos 2t + \frac{9}{5}e^{t}\sin$$

5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$, your had at $t=0$, $y=0$, $y'=2$

$$y'' - 6y' + 8y = e^{3t}$$

$$s^2y(s) - sy(0) - y'(0) - 6(sy(s) - y(0)) + 8(y(s)) = \frac{1}{s-3}$$

$$s^2y(s) - s\times 0 - 2 + 6(sy(s) - 0) + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 2 + 6sy(s) + 8y(s) = \frac{1}{s-3}$$

$$y(s)(s^2 + 6s + 8) = \frac{1}{-2(s-3)}$$

$$y(s) = \frac{1}{-2s+6} \times \frac{1}{s^2+6s+8}$$

$$y(s) = \frac{1}{(-2s+6)(s-4)(s-2)}$$

$$\frac{1}{(-2s+6)(s-4)(s-2)} = \frac{A}{(-2s+6)} + \frac{B}{(s-4)} + \frac{C}{(s-2)}$$

$$1 = A(s-4)(s-2) + B(-2s+6)(s-2) + C(-2s+6)(s-4)$$

if $s = 4$

$1 = B(-2)$, $B = -\frac{1}{2}$

if $s = 2$

$1 = C(2)(-2)$

$1 = -4C$, $C = -\frac{1}{4}$

if $s = 3$

$1 = A(-1)(1)$

$A = -1$

$$y(s) = \frac{-1}{-2} \cdot \frac{1}{s-3} - \frac{\frac{1}{2}}{s-4} - \frac{\frac{1}{4}}{(s-2)}$$

$$y(t) = L^{-1}y(s) = \frac{1}{2}e^{3t} - \frac{1}{2}e^{4t} - \frac{1}{4}e^{2t}$$