A3823 Tiyimtokunn Anelowu
15/engo4/062
Elect - Elect

ASSIGNMENT 5

$\frac{dy}{dt} + 3y = e^{-2t}$ at t 0, y 2

$L\{dy/dt\} = SY(s) - Y(0)$

$L\{3y\} = 3y(s)$

$L\{e^{-2t}\} = 1/s+2$

$sY(s) - y(0) - 3y(s) = \frac{1}{s+2}$

$sY(s) + 3y(s) - 2 = \frac{1}{s+2}$ $(s+3)y(s) = \frac{1}{s+2} + 2$

$(s+3)y(s) = \frac{1+2(s+2)}{s+2}$

$(s+3)y(s) = \frac{1+2s+4}{s+2}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

$2s+5 = As + 3A + Bs + 2B$

$A + B = 2 \times 3$ — (3)

$3A + 2B - 5 = 0$ — (4)

equ (3) - (4)

$B = 1$

Put $s = -2$ in equ

$A + 1 = 2 \quad A = 1$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$$

$y(t) = e^{-2t} + e^{-3t}$

2. $3\frac{dy}{dt} - 6y = \sin 2t$ at $t=0$, $y=1$

$L\{3\frac{dy}{dt}\} = 3sy(s) - y(0)$

$L\{6y\} = 6y(s)$

$L\{\sin 2t\} = \frac{2}{s^2+2^2}$

$$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+2^2}$$

$$3sy(s) - 6y(s) - 3y(0) = \frac{2}{s^2+2^2}$$

$$(3s-6)ys - 3 = \frac{2}{s^2+2^2} = (3s-6)ys = \frac{2}{s^2+2^2} + 3$$

$$(3s-6)y(s) = \frac{2}{(s+2)^2} + 3(s+2)^2$$

$$y(s) = \frac{2+3(s+2)^2}{(s+2)^2(3s-6)}$$

$$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{(3s-6)}$$

$2+3(s+1)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$

$2+3s^2+18s+12 = ABs^2 - 12A + 3Bs - 6B - Cs + 4sC + 4C$

$3A + C = 3$

$3B + 4C = 12$

$-12A - 6B + 4C = 14$

from equ (1)

$2A - 3 - C$

from equ (1)

$3B = 12 - 13 \quad 3B = -1 \quad B = \frac{1}{3}$

equ (1) $3A = 3 - C \quad 3A = 3 - \frac{13}{4} \quad A = -\frac{1}{12}$

$\frac{(2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{-1/12}{(s+2)} - \frac{1/3}{(s+2)} + \frac{13/4}{(2s-6)}$

$\therefore y(t) = -\frac{1}{12} e^{-3t} - \frac{1}{3} t e^{-4t} + \frac{13}{12} e^{2t}$

$\therefore y(t) = -\frac{1}{12} (e^{-2t} + 4t e^{-4t} - 13 e^{2t})$

B

3 $\frac{dy}{dt} - 4y = 8$ at $t - 6$, $y = 2$

$L\{\frac{dy}{dt}\} y = S(y(s)) - y(0)$

$L\{-4y\} = 4y(s)$

$L\{8\} y = 8/s$

$sy(s) - y(0) - 4y(s) = 8/s$

$sy(s) - 4y(s) - y(0) = 8/s$

$(s-4)y(s) = \frac{8}{s} + 2$

$(s-4)Y(s) = \frac{8}{s} + 2$

$(s-4)Y(s) = \frac{8+2s}{s}$

$Y = \frac{8+2s}{s(s-4)}$ $\qquad \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$8+2s = A(s-4) + Bs$

$8+2s = As - 4A + Bs$

$A+B = 2 \qquad -4A = 8 \qquad A = -2$

$B = 2+2 = 4$

$y(t) = -2 + 4e^{4t}$

1 $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ at $y = 2$ ...

$L\{d^2y/dt^2\} = s^2Y(s) - sy(0) + y(0)$

$L\{dy/dt\} = -2sY(s) + 2y(0)$

$L\{5y\} = 5Y(s)$

$L\{e^{2t}\} = 1/s+2$

$s^2Y(s) - sy(0) - y(0) - 2sY(s) + 2y(0) + 5Y(s)$

$s^2Y(s) - 2sY(s) + 5Y(s) - 2s + 4 = \frac{1}{s-2}$

$(s^2 - 2s + 5)Y(s) = \frac{1}{s-2} + 2s - 3$

$Y(s)(s^2 - 2s + 5) = \frac{(2s-3)(s-2)}{(s-2)}$

$Y(s) = \frac{1 + 2s^2 - s + 6}{(s-2)(s^2 - 2s + 5)}$

Scanned by CamScanner

$$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs+C}{s^2-2s+5}$$

$$2s^2-7s+7 = A(s^2-2s+5) + (Bs+C)(s-2)$$

$$2s^2-7s+7 = As^2-2As+5A+Bs^2-2Bs+Cs-2C$$

$$A+B=2$$

$$-2A-2B+C=7$$

$$5A-2C=7$$

from equation $B = 2-A$

from (2) $-2A-2(2-A)+C=-7 \quad --2A-4\cancel{A}\,\cancel{2A}+C=-7$

$C=3$

from (3) $5A-2(-3)=7 \quad 5A=7-6 \quad A=\frac{1}{5}$

$A+B=2 \quad \frac{1}{5}+B=2 \quad B=2-\frac{1}{5} \quad B=\frac{9}{5}$

$$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{\frac{1}{5}s}{s-2} + \frac{\frac{9}{5}(s-3)}{s^2-2s+5}$$

$$= \frac{\frac{1}{5}}{s+2} + \frac{\frac{9}{5}s}{s^2-2s+5} - \frac{3}{(s^2-3s+5)}$$

$$= \frac{\frac{1}{5}}{(s+2)} + \frac{9}{5}\left(\frac{s-1+1}{s^2-2s+5}\right) - \frac{3\times\frac{2}{2}}{s^2-2s+5}$$

$$= \frac{\frac{1}{5}}{s+2} + \frac{7}{5}\,\frac{(s-1+1)}{(s-1)^2+4} - \frac{3}{2}\left(\frac{2}{(s-1)^2+4}\right)$$

$$L^{-1}\left\{\frac{\frac{1}{5}}{s-2} + \frac{9}{5}\left\{\frac{s-1}{(s-1)^2+2^2} + \frac{1}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right\} - \frac{3}{2}\left\{\frac{2}{(s-1)\ldots}\right.\right.$$

$g(t) = \frac{1}{5} e^{2t} + \frac{7}{5} \{ e^t \cos 2t + \frac{1}{2} \{ e^t \sin 2t \} - \frac{3}{2} \{ e^t \sin 2t \}$

5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t = 0$, $y = 0$ $y' = 2$

$L\{\frac{d^2y}{dt^2}\} = s^2 y(s) - sy(0) - y'(0)$

$L\{6\frac{dy}{dt}\} = -6sy(s) + 6y(0)$

$L\{8y\} = 8y(s)$ $L\{e^{3t}\} = 1/s-3$

$s^2y(s) - sy(0) - y'(0) - 6sy(0) + 6y(0) + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - 6sy(s) + 8y(s) - 2 = \frac{1}{s-3}$

$y(s)(s^2 - 6s + 8) = \frac{1}{s-3} + 2$

$(s^2 - 6s + 8)y(s) = \frac{1 + 2(s-3)}{s-3}$

$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{s-3} + \frac{Bs+C}{s^2-6s+8}$

$2s - 5 = A(s^2 - 6s + 8) + (Bs + C)(s-3)$

$2s - 5 = As^2 - 6As + 8A + Bs^2 - 3Bs + Cs - 3C$

$A + B = 0$ $-6A - 3B + C = 0$

$-8A - 3C = 3$

$B = -A$ from eqn (1)

$-6A + 3A + C = 2$

$-3A + C = 2 \cdots (4) \times 3$

$8A - 3C = -5$

$9A - 3C = 6$

$8A - 3C = -5$

$A = -1 \quad , B = -1$

from $c = -2-3 = -1$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{s-3} + \frac{s-1}{s^2-6s+8}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{s-3} + \frac{s-1}{(s-2)(s-4)}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$s-1 = A(s-4) + B(s-2)$

$s-1 = As - 4A + Bs - 2B$

$A + B = 1 \quad \cdots (1) \times -4$

$-4A - 2B = -1 \quad \cdots (2) \times 1$

$-4A - 4B = -4$

$-4A - 2B = -1$

$-2B = -3$

$B = 3/2$

$A = -1/2$

$$\frac{s-1}{(s-2)(s-4)} = \frac{-1/2}{s-2} + \frac{3/2}{s-4}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{1}{s-3} + \left[\frac{-1/2}{s-2} + \frac{3/2}{s-4}\right]$$

$$y(t) = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t} = \frac{1}{2}\left[2e^{3t} + e^{2t} - 3e^{4t}\right]$$