

ELENDU DINE

10/ENG006/026

MECHANICAL ENGINEERING

ENG 281 : ASSIGNMENT 5

I $\frac{dy}{dt} + 3y = e^{-2t}$

$$y(t) + 3y(t) = e^{-2t}$$

$$L[y'(t)] = sY(s) - Y(0)$$

$$L[y(t)] = Y(s)$$

$$L^{-1}[e^{-2t}] = \frac{1}{s+2}$$

$$sY(s) - Y(0) + 3Y(s) = \frac{1}{s+2}$$

$$(s-3)Y(s) - 2 = \frac{1}{s+2}$$

$$(s-3)Y(s) = \frac{1}{s+2} + 2 = \frac{1+2s+4}{s+2} = \frac{2s+5}{s+2}$$

$$Y(s) = \frac{2s+5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$$

$$2(-2) + 5 = A(-2-3) \Rightarrow A = \frac{1}{-5}$$

$$2(3) + 5 = B(3+2) \Rightarrow B = \frac{11}{5}$$

$$Y(s) = \frac{-1}{s(s+2)} + \frac{11}{5(s-3)} = \frac{-1}{5} e^{-2t} + \frac{11}{5} e^{3t}$$

II $3\frac{dy}{dt} - 6y = \sin 2t = 3y'(t) - 6y(t)$

$$L[y'(t)] = sY(s) - Y(0)$$

$$L[y(t)] = Y(s)$$

$$L[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$

$$3s + Y(s) - 3(0) - 6Y(s) = \frac{2}{s^2+4}$$

$$Y(s) [3s-6] = \frac{2}{s^2+4} + 3 = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$$

$$Y(s) = \frac{3s^2 + 14}{(3s-6)(s^2+4)} = \frac{A}{3s-6} + \frac{B}{s^2+4}$$

$$3s^2 + 14 = A(s^2 + 4) + B(3s - 6)$$

$$4A - 6B = 14$$

$$B = -1/3$$

$$Y(s) = \frac{3}{3(s-2)} - \frac{1}{3(s^2+4)}$$

$$= e^{2t} - \frac{1}{6} \sin 2t$$

iii $\frac{dy}{dt} - 4y = 8$

$$y'(t) - 4y(t) = 8$$

$$y'(t) - 4y(t) = 8$$

$$sY(s) - Y(s) - 4Y(s) = 8/s$$

$$(s-4)Y(s) = 8/s + 2 = \frac{8+2s}{s}$$

$$Y(s) = \frac{2s+8}{s(s-4)}$$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s+8 = A(s-4) + B(s)$$

$$2(0)+8 = A(0-4) \quad (\therefore A = -2)$$

$$2(4)+8 = A(4-4) + B(4) \Rightarrow B = 4$$

$$L^{-1} \left[\frac{-2}{s} + \frac{4}{s-4} \right] = -2 + 4e^{4t}$$

iv $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$$y''(t) - 2y'(t) + 5y(t) = e^{2t}$$

$$L[y''(t)] = s^2 Y(s) - sY(s) - Y'(s)$$

$$L[y'(t)] = sY(s) - Y(s)$$

$$L[y(t)] = Y(s)$$

$$s^2 Y(s) - sY(s) - Y'(s) - 2sY(s) + 2Y(s) + 5Y(s) = \frac{1}{s-2}$$

$$[s^2 - 2s + 5] Y(s) + (2-s)Y(s) - Y'(s) = \frac{1}{s-2}$$

$$[s^2 - 2s + 5] Y(s) = \frac{1}{s-2} - (2-s) \cdot 2 \cdot 1 = \frac{1}{s-2} + 2s - 4 + 1$$

$$= \frac{1}{s-2} + 2s - 3 = \frac{1 + (2s-3)(s-2)}{(s-2)}$$

$$= \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$$

$$Y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$$

$$2s^2 - 7s + 7 = \frac{A}{s-2} + \frac{B}{s^2 - 2s + 5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$$

$$A = 2$$

$$-2A + B = -7$$

$$B = -3$$

$$Y(s) = \frac{2}{s-2} - \frac{3}{s^2 - 2s + 5}$$

$$= 2e^{2t} - \frac{1}{5} + \sin 2t$$

$$v \quad \frac{d^2y}{dt^2} - 6 \frac{dy}{dt} + 8y = e^{3t}$$

$$s^2 Y(s) - 6Y(s) - Y'(s) - 6sY(s) + 6Y(s) + 8Y(s) = \frac{1}{s-3}$$

$$[s^2 - 6s + 8] Y(s) = \frac{1}{s-3} + 2 = \frac{1 + 2s - 6}{s-3}$$

$$= \frac{2s - 5}{s-3}$$

$$Y(s) = \frac{2s - 5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$2s - 5 = A[(s-2)(s-4)] + B[(s-3)(s-4)] + C[(s-3)(s-2)]$$

$$2s - 5 = A(s^2 - 6s + 8) + B(s^2 - 7s + 12) + C(s^2 - 5s + 6)$$

$$2(3) - 5 = A(3-2)(3-4) \Rightarrow A = 4$$

$$2(4) - 5 = C(4-3)(4-2) \Rightarrow C = \frac{3}{2}$$

$$-6A - 7B - 5C = 7$$

$$-6(4) - 7B - 5(\frac{3}{2}) = 7$$

$$-7B = 2 + \frac{15}{2} - 6 = \frac{4 + 15 - 12}{2} = \frac{7}{2} \Rightarrow B = -\frac{1}{2}$$

$$L^{-1} \left[\frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)} \right]$$

$$= -e^{3t} - \frac{1}{2} e^{2t} + \frac{3}{2} e^{4t}$$