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Dept: Petroleum Engineering
Course: ENG 381 Assignment 5

1) $dy/dt + 3y = e^{-2t}$

$y(t) + 3y(t) = e^{-2t}$

$L[y'(t)] = sY(s) - Y(s)$

$L[y(t)] = Y(s), L^{-1}[e^{-2t}] = \frac{1}{s+2}$

$sY(s) - Y(s) + 3Y(s) = \frac{1}{s+2}$

$(s-3)Y(s) - 2 = \frac{1}{s+2}$

$(s-3)Y(s) = \frac{1}{s+2} + 2 = \frac{1+2s+4}{s+2} = \frac{2s+5}{s+2}$

$Y(s) = \frac{2s+5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$

$2(-2)+5 = A(-2-3) \Rightarrow A = \frac{1}{-5}$

$2(3)+5 = B(3+2) \Rightarrow B = \frac{11}{5}$

$Y(s) = \frac{-1}{5(s+2)} + \frac{11}{5(s-3)} = -\frac{1}{5}e^{-2t} + \frac{11}{5}e^{3t}$

ii) $3\frac{dy}{dt} - 6y = \sin 2t = 3y'(t) - 6y(s)$

$L[y'(t)] = sY(s) - Y(s)$

$L[y(t)] = Y(s)$

$L[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$

$3sY(s) - 3Y(s) - 6Y(s) = \frac{2}{s^2+4}$

$Y(s)[3s-6] = \frac{2}{s^2+4} + 3 = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$

$Y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{As+B}{s^2+4} + \frac{C}{3s-6}$

$3s^2+14 = A(s^2+4) + B(3s-6)$

$4A - 6B = 14$

$-6B = 14 - 12$

$B = -1/3$

$Y(s) = \frac{3}{[3][s-2]} - \frac{1}{3[s^2+4]}$

$= e^{2t} - \frac{1}{6}\sin 2t$

iii) $\frac{dy}{dt} - 4y = 8$

$y'(t) - 4Y(s) = 8$

$Y'(t) - 4y(s) = 8$

$sY(s) - Y(s) - 4y(s) = \frac{8}{s}$

$(s-4)Y(s) = \frac{8}{s} + 2 = \frac{8+2s}{s}$

$Y(s) = \frac{2s+8}{s(s-4)}$

$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$2s+8 = A(s-4) + B(s)$

$2(0)+8 = A(0-4) \Rightarrow A = -2$

$2(4)+8 = A(4-4) + B(4)$

$B = 4$

$L^{-1}[-2/s + 4/s-4] = -2 + 4e^{4t}$

iv) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$y''(t) - 2y'(t) + 5y(t) = e^{2t}$

$L[y''[t]] = s^2Y(s) - sY(s) - y'(0)$

$L[y'(t)] = sY(s) - Y(s)$

$L[Y(t)] = Y(s)$

$s^2Y(s) - sY(s) - y'(s) - 2sY(s) + 2Y(s) + 5Y(s) = \frac{1}{s-2}$

$[s^2-2s+5)Y(s)+(2-0)y_0-y'(0)$

$=\frac{1}{s-2}$

$[s^2-2s+5]Y(s)=\frac{1}{s-2}-2(2-5)+1$

$=\frac{1}{s-2}+2s-4+1=\frac{1}{s-2}+2s-3$

$=\frac{1+(2s-3)(s-2)}{(s-2)}=\frac{1+2s^2-4s-3s+6}{s-2}$

$Y(s)=\frac{2s^2-7s+7}{(s-2)(s^2+2s+5)}$

$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)}=\frac{A}{s-2}+\frac{B}{s^2-2s+5}$

$2s^2-7s+7=A(s^2-2s+5)+B(s-2)$

$A=2$

$-2A+B=-7$

$B=-7+4=-3$

$=\frac{2}{s-2}-\frac{3}{s-2s+5}$

$2e^{2t}-\frac{7}{3}\sin 2t$

V) $\frac{d^2y}{dt^2}-6\frac{dy}{dt}+8y=e^{3t}$

$s^2Y(s)-sY(s)-Y(0)-6sY(s)+6Y(0)+8Y(s)=\frac{1}{s-3}$

$[s^2-6s+8]Y(s)+(6-5)Y(0)-Y(0)=\frac{1}{s-3}$

$[s^2-6s+8]Y(s)=\frac{1}{s-3}+2=$

$\frac{1+2s-5}{s-3}=\frac{2s-5}{s-3}$

$Y(s)=\frac{2s-5}{(s-3)(s-2)(s-4)}=\frac{A}{s-3}+\frac{B}{s+2}+\frac{C}{s-4}$

$2s-5=A(s-2)(s-4)+B(s-3)(s-4)+[(3-3)(s-2)]$

$2s-5=A(s^2-6s+8)+B(s^2-7s+12)+(Cs^2-5s+6)$

$2(3)-5=A(3-2)(3-4)$

$2(4)-5=C(4-3)(4-2)$

$-6A-7B-5C=7$

$-6C-3-7(B)-5(3/2)=3$

$-7B=2+15-6=\frac{4+15-12}{2}=7/2$

$B=-1/2$

$L^{-1}\left[\frac{-1/2}{s-3}-\frac{1}{2(s-2)}+\frac{3/2}{2(s-4)}\right]$

$=e^{3t}-\frac{1}{2}e^{2t}+\frac{3}{2}e^{4t}$