AMOO EMMANUEL
15/ENG06/011

ENG 381.

1) $\frac{dy}{dt} + 3y = e^{-2t}$, given that at $t = 0$, $y = 2$.

$sY_{(s)} = y_{(0)} + 3Y_{(s)} = \frac{1}{s+2}$.

$sY_{(s)} - 2 + 3Y_{(s)} = \frac{1}{s+2}$

$sY_{(s)} + 3Y_{(s)} = \frac{1}{s+2} + 2$

$Y_{(s)}(s+3) = \frac{1 + 2s + 4}{s+2} = \frac{2s+5}{s+2}$

$Y_{(s)} = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$2s+5 = A(s+3) + B(s+2)$

let $s = -3$.

$-1 = -B$

$B = 1$

let $s = -2$

$1 = A$

$Y_{(s)} = L^{-1}\left[\frac{1}{(s+2)} + \frac{1}{(s+3)}\right]$

$L^{-1}\{Y_{(s)}\} = L^{-1}[1/s+2] + L^{-1}\{1/s+3\}$.

$y = e^{-2t} + e^{-3t}$.

2. $3\,dy/dt - 6y = \sin 2t$ given that at $t = 0$; $y = 1$.

$L\{3\,dy/dt\} = 3\{sY_{(s)} - Y_{(0)}\}$.

$L\{-6y\} = -6\{Y_{(s)}\}$.

$L\{\sin 2t\} = 2/s^2+2^2$.

$3sY_{(s)} - 3Y_{(0)} - 6Y_{(s)} = 2/s^2+4$.

at $t = 0$; $y = 1$.

$3sY_{(s)} - (3\times1) - 6Y_{(s)} = 2/s^2+4$.

$$3sY(s) - 3 - 6Y(s) = 2/(s^2+4)$$

$$Y(s)\{3s-6\} = 2/(s^2+4) + 3/1 = \frac{2+3(s^2+4)}{s^2+4}$$

$$Y(s)\{3s-6\} = \frac{2+3s^2+12}{(s^2+4)} = \frac{3s^2+14}{(s^2+4)}.$$

$$Y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)}.$$

using partial fraction.

$$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{(s^2+4)} + \frac{C}{(3s-6)}$$

$$\left.\frac{3s^2+14}{s^2+4}\right|_{s=2} = \frac{3(2)^2+14}{2^2+4} = \frac{13}{4}.$$

$$3s^2+14 = (A+Bs)(3s-6) + C(s^2+4)$$

$$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C$$

comparing co-efficient.

$$3 = 3B + C$$

$$3 = 3B + 13/4$$

$$3A - 6B = 0$$

$$3A = 6B$$

$$3A = 6 \times -1/12$$

$$A = -1/6.$$

$$3B = -1/4.$$

$$B = -1/12.$$

$$Y(s) = \frac{-1/6 - \{-1/12\}s}{s^2+4} + \frac{13/4}{3s-6}$$

$$= \frac{-1/6}{s^2+4} - \frac{1/12\, s}{s^2+4} + \frac{13/4}{3s-6}$$

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$$= \frac{1}{6} \cdot \frac{1}{s^2+2^2} - \frac{1}{12} \cdot \frac{s}{s^2+2^2} + \frac{13}{4} \cdot \frac{1}{3(s-2)}$$

$$= \frac{-1}{6} \cdot \frac{1}{2}\left\{\frac{2}{s^2+2^2}\right\} - \frac{1}{12}\left\{\frac{s}{s^2+2^2}\right\} + \frac{13}{12}\left\{\frac{1}{s-2}\right\}$$

$$y(t) = \frac{-1}{12}\sin 2t - \frac{1}{12}\cos 2t + \frac{13}{12}e^{2t}.$$

$$y(t) = \frac{1}{12}\left\{-\sin 2t - \cos 2t + 13e^{2t}\right\}.$$

$$y(t) = \frac{1}{12}\left\{13e^{2t} - \cos 2t - \sin 2t\right\}.$$

3. $dy/dt - 4y = 8$ at $t=0$; $y=2$.

$y' - 4y = 8.$

$sy(s) - y(0) - 4y(s) = 8/s.$

Applying the condition.

$y(s)\{s-4\} = 8/s + 2/1 = 8+2s/s.$

$$y(s) = \frac{8+2s}{s\{s-4\}} = \frac{A}{s} + \frac{B}{s-4}.$$

$$\left.\frac{8+2s}{s-4}\right|_{s=0} = \frac{8}{-4} = -2.$$

$$\left.\frac{8+2s}{s}\right|_{s=4} = \frac{8+2(4)}{4} = 4.$$

$$y(s) = \frac{-2}{s} + \frac{4}{s-4}$$

$$y(t) = -2 + 4e^{4t}$$

4. $d^2y/dx^2 - 2\,dy/dx + 5y = e^{2t}.$

at $t=0$; $y=2$; $y'=1$.

$y'' - 2y' + 5y = e^{2t}.$

$\{s^2y(s) - sy(0) - y'(0)\} - 2\{sy(s) - y(0)\} + 5y(s) = 1/s-2.$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = 1/s-2.$

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$$Y(s)\,[s^2-2s+5] = \frac{1}{s-2} + \frac{2s}{1} - \frac{3}{1} = \frac{1+2s(s-2)-3(s-2)}{s-2}$$

$$Y(s)\,[s^2-2s+5] = \frac{1+2s^2-4s-3s+6}{s-2} = \frac{2s^2-7s+7}{s-2}.$$

$$Y(s) = \frac{2s^2-7s+7}{(s-2)(s^2-2s+5)}$$

Using partial partial:

$$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs+C}{s^2-2s+5}.$$

$$(A) = \left.\frac{2s^2-7s+7}{s^2-2s+5}\right|_{s=2} = \frac{2(2)^2-7(2)+7}{2^2-2(2)+5} = \frac{1}{5}.$$

$$2s^2-7s+7 = A[s^2-2s+5] + (Bs+C)(s-2).$$

$$2s^2-7s+7 = As^2-2As+5A+Bs^2-2Bs+Cs-2C.$$

$$2 = A+B$$

$$2 = 1/5 + B.$$

$$B = 9/5.$$

$$7 = 5A-2C.$$

$$7 = 5(1/5)-2C.$$

$$7-1 = -2C.$$

$$C = -3.$$

$$Y(s) = \frac{1/5}{s-2} + \frac{9/5(s-3)}{s^2-2s+5} = \left\{\frac{1}{5}\cdot\frac{1}{s-2}\right\} + \frac{9/5\, s}{[s+1]^2+4} - \frac{3}{(s+1)^2}$$

$$Y(s) = \left[1/5 \cdot 1/(s-2)\right] + 9/5 \cdot \frac{s+1}{(s+1)^2+2^2} - \frac{1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$$

$$Y(s) = \left[1/5 \cdot 1/(s-2)\right] + 9/5 \cdot \frac{s+1}{(s+1)^2+2^2} - \frac{4\cdot 2/2}{(s+1)^2+2^2}$$

$$Y(s) = \left\{1/5 \cdot 1/(s-2)\right\} + \left\{9/5 \cdot \frac{s+1}{(s+1)^2+2^2}\right\} - 2\cdot\frac{2}{(s+1)^2+2^2}$$

$y(t) = \frac{1}{5} e^{2t} + \frac{9}{5} e^{-t} \cos 2t - 2e^{-t} \sin 2t.$

$y(t) = \frac{1}{5} \{ e^{2t} + 9e^{-t} \cos 2t + 10e^{-t} \sin 2t \}.$

$= \frac{1}{5} [e^{2t} + e^{-t} \{ 9 \cos 2t - 10 \sin 2t \}]$ //.

5. $\frac{d^2y}{dt^2} - 6 \frac{dy}{dt} + 8y = e^{3t}$

at $t = 0$, $y = 0$, $y' = 2$.

$y'' - 6y' + 8y = e^{3t}$

$s^2 y(s) - s y(0) - y'(0) - 6 \{ s y(s) - y(0) \} + 8 y(s) = \frac{1}{s-3}.$

At $t = 0$; $y = 0$; $y' = 2$.

$s^2 y(s) - 2 - 6s\, y(s) + 8 y(s) = \frac{1}{s-3}.$

$y(s) \{ s^2 - 6s + 8 \} = \frac{1}{s-3} + \frac{2}{1}.$

$y(s) \{ s^2 - 6s + 8 \} = \frac{1 + 2(s-3)}{s-3}.$

$y(s) \{ s^2 - 6s + 8 \} = \frac{1 + 2s - 6}{s-3}$

$y(s) = \frac{2s - 5}{(s-3)(s^2 - 6s + 8)} = \frac{2s - 5}{(s-3)(s-2)(s-4)}$

Using partial fraction.

$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{(s-3)} + \frac{B}{(s-2)} + \frac{C}{(s-4)}$

A: $\left. \frac{2s-5}{(s-2)(s-4)} \right|_{s=3} = \frac{2(3)-5}{(3-2)(3-4)} = -1.$

B; $\left. \frac{2s-5}{(s-3)(s-4)} \right|_{s=2} = \frac{2(2)-5}{(2-3)(2-4)} = \frac{-1}{2}.$

C; $\left. \frac{2s-5}{(s-3)(s-2)} \right|_{s=4} \quad \frac{2(4)-5}{(4-3)(4-2)} = \frac{3}{2}.$

$y(s) = \left\{ \frac{-1}{s-3} \right\} - \left\{ \frac{1}{2} \cdot \frac{1}{s-2} \right\} + \left\{ \frac{3}{2} \cdot \frac{1}{s-4} \right\}.$

$y(t) = e^{3t} - \frac{1}{2} e^{2t} + \frac{3}{2} e^{4t}$ //.