

$$3s^2 + 14 = A(s^2 + 4) + B(2s - 6)$$

$$4A - 6B = 14$$

$$-6B = 14 - 12$$

$$B = \frac{1}{3}$$

$$Y(s) = \frac{3}{3(s^2 + 4)} = \frac{1}{s^2 + 4}$$

$$= e^{2t} - \frac{1}{6} \sin 2t$$

iii.) $\frac{dy}{dt} - 4y = 8$

$$Y'(s) - 4Y(s) = 8$$

$$Y'(s) - 4Y(s) = 8$$

$$sY(s) - Y(0) - 4Y(s) = \frac{8}{s}$$

$$(s-4)Y(s) = \frac{8}{s} + 2 = \frac{8+2s}{s}$$

$$Y(s) = \frac{2s+8}{s(s-4)}$$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s+8 = A(s-4) + B(s)$$

$$2(0)+8 = A(0-4) \quad (\therefore A = -2)$$

$$2(4)+8 = A(4-4) + B(4)$$

$$B = 4$$

$$L\left[\frac{-2}{s} + \frac{4}{s-4}\right] = -2 + 4e^{4t}$$

iv.) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$$Y''(s) - 2Y'(s) + 5Y(s) = \frac{1}{s-2}$$

$$L[Y''(s)] = s^2 Y(s) - sY(0) - Y'(0)$$

$$L[Y'(s)] = sY(s) - Y(0)$$

$$L[Y'(t)] = Y(s)$$

$$s^2 Y(s) - s Y(0) - Y'(0) - 2s Y(s) + 2 Y(s) + 5 Y(s) = \frac{1}{s-2}$$

$$[s^2 - 2s + 5] Y(s) + (2-s) Y_0 - Y'(0) = \frac{1}{s-2}$$

$$[s^2 - 2s + 5] Y(s) = \frac{1}{s-2} - (2-s)2 + 1$$

$$= \frac{1}{s-2} + 2s - 4 + 1$$

$$= \frac{1}{s-2} + 2s - 3$$

$$= \frac{1 + (2s-3)(s-2)}{(s-2)}$$

$$= \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$$

$$Y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$$

$$2s^2 - 7s + 7 = \frac{A}{s-2} + \frac{B}{s^2 - 2s + 5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$$

$$A = 2$$

$$-2A + B = -7$$

$$B = -7 + 4$$

$$= -3$$

$$Y(s) = \frac{2}{s-2} - \frac{3}{s^2 - 2s + 5}$$

$$= 2e^{2t} - \frac{7}{3} + \sin 2t$$

$$v) \frac{d^2 y}{dt^2} - 6 \frac{dy}{dt} + 8y = e^{3t}$$

$$s^2 Y(s) - s Y(0) - Y'(0) - 6s Y(s) + 6 Y(s) + 8 Y(s) = \frac{1}{s-3}$$

$$s Y(s) = \frac{1}{s-3}$$

$$\begin{aligned}
 \frac{1}{s^2 - 6s + 8} Y(s) &= \frac{1}{s-3} + \frac{2}{s-4} = \frac{1+2s-6}{s-3} \\
 &= \frac{2s-5}{s-3}
 \end{aligned}$$

$$Y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$2s-5 = A[(s-2)(s-4)] + B[(s-3)(s-4)] + C[(s-3)(s-2)]$$

$$2s-5 = A(s^2 - 6s + 8) + B(s^2 - 7s + 12) + C(s^2 - 5s + 6)$$

$$2(3) - 5 = A(3-2)(3-4) \Rightarrow A = 4$$

$$2(4) - 5 = C(4-3)(4-2) \Rightarrow C = \frac{3}{2}$$

$$-6A - 7B - 5C = 7$$

$$-6(-1) - 7B - 5\left(\frac{3}{2}\right) = 7$$

$$-7B = 7 + \frac{15}{2} - 6 = \frac{4+15-12}{2} = \frac{7}{2}$$

$$\Rightarrow B = -\frac{1}{2}$$

$$\begin{aligned}
 L^{-1} \left[\frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)} \right] \\
 = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}
 \end{aligned}$$

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15/ENG04/058

ELECTRICAL/ELECTRONIC ENGR

1) $\frac{dy}{dt} + 3y = e^{-2t}$

$$y(t) + 3y(t) = e^{-2t}$$

$$L[y'(t)] = SY(s) - Y(0)$$

$$L[y(t)] = Y(s)$$

$$L^{-1}[e^{-2t}] = \frac{1}{s+2}$$

$$SY(s) - Y(0) + 3Y(s) = \frac{1}{s+2}$$

$$(s-8)Y(s) - 2 = \frac{1}{s+2}$$

$$(s-3)Y(s) = \frac{1}{s+2} + 2 = \frac{1+2s+4}{s+2} = \frac{2s+5}{s+2}$$

$$Y(s) = \frac{2s+5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$$

$$2(-2)+5 = A(-2-3) \Rightarrow A = \frac{1}{-5}$$

$$2(3)+5 = B(3+2) \Rightarrow B = \frac{11}{5}$$

$$Y(s) = \frac{-1}{5(s+2)} + \frac{11}{5(s-3)} = \frac{-1}{5}e^{-2t} + \frac{11}{5}e^{3t}$$

ii) $3\frac{dy}{dt} - 6y = \sin 2t = 3y'(t) - 6y(t)$

$$L[y'(t)] = SY(s) - Y(0)$$

$$L[y(t)] = Y(s)$$

$$L[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$

$$3s + Y(s) - 3Y(0) - 6Y(s) = \frac{2}{s^2+4}$$

$$Y(s)[3s-6] = \frac{2}{s^2+4} + 3 = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$$

$$Y(s) = \frac{3s^2+14}{(3s-6)(s^2+4)} = \frac{A}{3s-6} + \frac{B}{s^2+4}$$