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15/ENG03/033

ENG 381

1)  $dy + 3y = e^{-2t}$

$t=0, y=2$

$$8y(s) + y(s) + 3y(s) = \frac{1}{s+2}$$

$$y(s)(s+3) + 2 = \frac{1}{s+2}$$

$$y(s)(s+3) = \frac{1}{s+2} + 2$$

$$y(s)(s+3) = \frac{1+2s+4}{s+2}$$

$$y(s) = \frac{2s+5}{(s+2)(s+3)}$$

$L^{-1} \left\{ \frac{2s+5}{(s+2)(s+3)} \right\} = y(s)$

$$y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$$

$$2s+5 = A(s+3) + B(s+2)$$

when  $s=-2$

$$-4+5 = A(1) + 0$$

$$A=1$$

when  $s=-3$

$$-6+5 = A(0) + B(-1)$$

$$\frac{-B}{-1} = \frac{-1}{-1}$$

$$B=1$$

$$y(s) = L^{-1} \left[ \frac{1}{s+2} - \frac{1}{s+3} \right]$$

$$y(s) = e^{-2t} + 3e^{-3t}$$

$$2) \quad 3 \frac{dy}{dt} - 6y = \sin 2t$$

at  $t=0, y=1$

$$3(sy(s) - y(0)) - 6y(s) = \frac{2}{s^2+4}$$

$$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+4}$$

$$3y(s)(s-2) - 3 = \frac{2}{s^2+4}$$

$$3y(s)(s-2) = \frac{2}{s^2+4} + 3$$

$$\frac{2 + 3s^2 + 12}{s^2+4}$$

$$y(s) = \frac{3s^2 + 14}{3(s-2)(s^2+4)}$$

$$= \frac{3s^2 + 14}{(3s-6)(s^2+4)}$$

$$y(s) = L^{-1} \left[ \frac{3s^2 + 14}{(3s-6)(s^2+4)} \right]$$

$$\frac{A}{(3s-6)} + \frac{Bs+C}{(s^2+4)}$$

$$3s^2 + 14 = A(s^2+4) + Bs + C(3s-6)$$

at  $s=2$

$$12 + 14 = A(8) + (Bs+C)(0)$$

$$\frac{26}{8} = \frac{8A}{8}$$

$$A = \frac{13}{4}$$

Using the method of coefficient =

$$3 = A + B \quad \text{--- (1)}$$

$$3 = \frac{13}{4} + 3B$$

$$3B = 3 \cdot \frac{13}{4}$$

$$3B = \frac{12-13}{4}$$

$$3B = -1/4, \quad B = -1/12$$

$$14 = 4A - 6C$$

$$14 = 4 \times \frac{13}{4} - 6C$$

$$14 = 13 - 6C$$

$$1 = -6C$$

$$C = -1/6$$

$$y(s) = L^{-1} \left[ \frac{13}{14} \times \frac{1}{s-2} + \frac{(-1/12s - 1/6)}{s^2+4} \right]$$

$$= L^{-1} \left[ \frac{13}{12} \times \frac{1}{s-2} - \frac{1s}{12} \times \frac{1}{s^2+4} - \frac{1}{6} \times \frac{1}{s^2+4} \right]$$

$$= L^{-1} \left[ \frac{13}{12} \times \frac{1}{s-2} \right] + L^{-1} \left[ \frac{-1s}{12} \times \frac{1}{s^2+4} \right] + L^{-1} \left[ \frac{1}{6 \times 2} \times \frac{2}{s^2+4} \right]$$

$$= \frac{13}{12} e^{2t} - \frac{1}{12} \cos 2t + \frac{1}{12} \sin 2t$$

3)  $dy - 4y = 8$

at  $t=0, y=2$

$$3y(s) - y(0) - 4y(s) = \frac{8}{s}$$

$$5y(s) - 2 - 4y(s) = \frac{8}{s}$$

$$y(s)(s-4) = \frac{8}{s} + 2$$

$$y(s)(s-4) = \frac{8+2s}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)}$$

$$y(s) = L^{-1} \frac{8+2s}{s(s-4)} = L^{-1} \left[ \frac{A}{s} + \frac{B}{s-4} \right]$$

$$\frac{8+2s}{s(s-4)} = \frac{A(s-4) + B(s)}{s(s-4)}$$

$$8 + 2s = A(s-4) + B(s)$$

When  $s=4$

$$8 + 8 = A(0) + 4B$$

$$B=4$$

When  $s=0$

$$8 = -4A + 0$$

$$\frac{-4A}{4} + \frac{8}{4}$$

$$A = -2$$

$$y(s) = L^{-1} \left[ \frac{-2}{s} + \frac{4}{s-4} \right]$$

$$y(s) = -2 + 4e^{4t}$$

$$4) \frac{d^2 y}{dt^2} - 2 \frac{dy}{dt} + 5y = e^{2t}$$

$$s^2 y(s) - 5y(0) - y'(0) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$$

at  $t=0, y=2, y'=1$

$$s^2 y(s) - s(2) - 1 - 2(sy(s) - 2) + 5y(s)$$

$$s^2 y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s)$$

$$s^2 y(s) - 2sy(s) + 5y(s) - 2s + 3 = \frac{1}{s-2}$$

$$y(s) (s^2 - 2s + 5) - 2s + 3 = \frac{1}{s-2}$$

$$y(s) (s^2 - 2s + 5) = \frac{1}{s-2} + \frac{2s}{1} + \frac{3}{1}$$

$$= \frac{1 + 2s^2 - 4 + 3s - 6}{s-2}$$

$$y(s) (s^2 - 2s + 5) = \frac{2s^2 + 3s - 9}{s-2}$$

$$y(s) = \frac{2s^2 - 13s - 9}{(s-2)(s^2 - 2s + 5)}$$

$$L^{-1} \left[ \frac{2s^2 + 3s - 9}{(s-2)(s^2 - 2s + 5)} \right] = \left[ \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5} \right]$$

$$2s^2 + 3s - 9 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$$

$$a + s = 2$$

$$8 + 6 - 9 = A(4 - 8 + 5) + 0$$

$$A = 5$$

Using the method of coefficient

$$2 = A + B$$

$$2 = 5 + B$$

$$B = -3$$

$$3 = -2A - 2B + C$$

$$3 = -2(5) - 2(-3) + C$$

$$3 = -10 + 6 + C$$

$$3 = -4 + C$$

$$C = -7$$

$$Y(s) = \frac{5}{s-2} + \frac{(-3s-7)}{s^2-2s+5}$$

$$= L^{-1} \left[ \frac{5}{s-2} \right] - L^{-1} \left[ \frac{3s}{(s-1)^2 + 4} \right]$$

$$= L^{-1} \left[ \frac{7}{(s-1)^2 + 4} \right]$$

$$= L^{-1} \left[ \frac{5}{s-2} \right] - L^{-1} \left[ \frac{3 \times 5}{(s-1)^2 + 4} \right]$$

$$= L^{-1} \left[ \frac{7}{2} \times \frac{2}{(s-1)^2 + 4} \right]$$

$$= 5e^{2t} - 3e^t \cos 2t - \frac{7}{2} e^t \sin 2t$$

5)  $\frac{d^2y}{dt^2} - \frac{6dy}{2t} + 8y = e^{3t}$

at  $t=0, y=0, y'=2$

$$= 5^2 y(s) - 5y(0) - y'(0) - 6(sy(s) - y(s)) + 8y(s)$$

$$= 5^2 y(s) - 0 - 2 - 6sy(s) + 6(0) + 8y(s) = \frac{1}{s-3}$$

$$= 5^2 y(s) - 2 - 6sy(s) + 8y(s) = \frac{1}{s-3}$$

$$= y(s) (s^2 - 6s + 8) = \frac{1}{s-3} + \frac{2}{1}$$

$$= y(s) (s^2 - 6s + 8) = \frac{1 + 2s + 6}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$y(s) = L^{-1} \left[ \frac{2s-5}{(s-3)(s-4)(s-2)} \right]$$

$$\frac{2s-5}{(s-3)(s-4)(s-2)} = \frac{A}{s-3} + \frac{B}{s-4} + \frac{C}{s-2}$$

$$\frac{2s-5}{(s-3)(s-4)(s-2)} = A(s-4)(s-2) + B(s-3)(s-2) + \frac{C(s-3)(s-4)}{(s-3)(s-2)(s-4)}$$

$$2s-5 = A(s-4)(s-4) + B(s-3)(s-2) + C(s-3)(s-4)$$

at  $s=2$

$$4-5 = A(0) + B(0) + C(-1)(-2)$$

$$-1 = 2C$$

$$C = -\frac{1}{2}$$

$$s=3$$

$$6-5 = A(-1)(1) + B(0) + C(0)$$

$$1 = -A \therefore A = -1$$

$$\text{at } s=4$$

$$8-5 = A(0) + B(2)(1) + C(0)$$

$$3 = 2B$$

$$B = \frac{3}{2}$$

$$y(s) = L^{-1} \left[ \frac{-1}{s-3} + \frac{3}{2} \times \frac{1}{s-4} - \frac{1}{2} \times \frac{1}{s-2} \right]$$

$$y(s) = -e^{3t} + \frac{3}{2}e^{4t} - \frac{1}{2}e^{2t}$$