ENG381 Assignment

$\frac{dy}{dt} + 3y = e^{-2t}$ given that at $t=0, y=2$

$L\{dy/dt\} = sy(s) - y(0)$

$L\{3y\} = 3y$

$L\{e^{-2t}\} = \frac{1}{s+2}$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) + 3y(s) - 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1+2(s+2)}{(s+2)}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

$2s+5 = As+3A+Bs+2B$

$A+B = 2 \times 3$

$3A+2B = 5 \times 1$

$3A+3B = 6$

$3A+2B = 5$

$B = 1$

from eqn (1)

$A+1 = 2$

$A = 2-1 = 1$

$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$

2) $3\frac{dy}{dt} - 6y = \sin 2t$ given that $t=0$, $y=1$

$L\{3\frac{dy}{dt}\} = 3\{sy(s) - y(0)\}$

$L\{-6y\} = -6y(s)$

$L\{\sin 2t\} = \frac{2}{s^2+2^2}$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+2^2}$

$3sy(s) - 6y(s) - 3 = \frac{2}{(s+2)^2}$

$y(s)(3s-6) = \frac{2}{(s+2)^2} + 3$

$y(s)(3s-6) = \frac{2+3(s+2)^2}{(s+2)^2}$

$y(s) = \frac{2+3(s+2)^2}{(s+2)^2(3s-6)}$

$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{(3s-6)}$

$2+3(s+2)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)$

$2+3s^2+12s+12 = A3s^2 - A12 + 3Bs - 6B + Cs^2 + 4Cs + 2C$

$3A+C = 3 \quad \ldots (1)$

$3B+4C = 12 \quad \ldots (2)$

$-12A - 6B + 4C = 14 \quad \ldots (3)$

From (1)

$3A = 3-C$

$A = \frac{3-C}{3}$

$3B + 4C = 12$

$-12\left(\frac{3-C}{3}\right) - 6B + 4C = 14$

$-12 + 4C - 6B + 4C = 14 \times -6$

$-6B + 8C = 26$

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$-18B - 24C = -72$

$+ -18B + 24C = 78$

$-48C = -150$

$C = +\frac{25}{8}$

From (2)

$3B = 12 - 4\left(\frac{25}{8}\right)$

$B = -1/6$

from (1)

$8A = 3 - C$

$A = 3 - \left(\frac{25}{8}\right)$

$A = -1/24$

$\frac{2+3(s+2)^2}{(s+2)^2(s^2-6)} = \frac{-1/24}{(s+2)} + \frac{1/6}{(s+2)^2} + \frac{25/8}{(s^2-6)}$

$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{-1/24}{(s+2)} - \frac{1/6}{(s+2)^2} + \frac{25/8}{(s^2-6)}\right\}$

$y = -\frac{1}{24}e^{-2t} - \frac{1}{6}te^{-4t} + \frac{25}{24}e^{3t}$

$y = -\frac{1}{6}\left\{\frac{1}{4}e^{-2t} + te^{-4t} + \frac{25}{4}e^{3t}\right\}$

8) $\frac{dy}{dt} - 4y = 8$ given that $t=0$, $y=2$

$L\{dy/dt\} = sy(s) - y(0)$

$L\{-4y\} = -4y(s)$

$L\{8\} = 8/s$

$sy(s) - y(0) - 4y(s) = 8/s$

$sy(s) - 4y(s) - y(0) = 8/s$

$y(s)(s-4) - 2 = \frac{8}{s} + 2$

$y(s)(s-4) = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)}$

$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$8+2s = A(s-4) + Bs$

$A + B = 2$

$-4A = 8$

$A = -2$

$B = 2+2$

$B = 4$

$\frac{8+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{(s-4)}$

$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{2}{s} + \frac{4}{s-4}\right\}$

$y = -2 + 4e^{4t}$

4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ given that $t=0$, $y=2$, $y=1$

$L\left\{\frac{d^2y}{dt^2}\right\} = s^2y(s) - sy(0) - y'(0)$

$L\left\{-2\frac{dy}{dt}\right\} = -2sy(s) + 2y(0)$

$L\{5y\} = 5y(s)$

$L\{e^{2t}\} = \frac{1}{s-2}$

$s^2y(s) - 5y(s) - y'(0) - 2sy(s) + 2y(0) + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2sy(s) + 5y(s) - 2s - 1 + 4 = \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 3$

$y(s)(s^2 - 2s + 5) = \frac{(2s-3)(s-2)}{(s-2)}$

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$$Y(s) = \frac{1 + 2s^2 - s + 6}{(s-2)(s^2 - 2s + 5)}$$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{A}{(s-2)} + \frac{Bs + C}{(s^2 - 2s + 5)}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$$

$$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

$$A + B = 2 \quad \cdots (1)$$

$$-2A - 2B + C = -7 \quad \cdots (2)$$

$$5A - 2C = 7 \quad \cdots (3)$$

from (1)

$$B = 2 - A$$

from (2)

$$-2A - 2(2 - A) + C = -7$$

$$-2A - 4 + 2A + C = -7$$

$$C = -3$$

from (3)

$$5A - 2(-3) = 7$$

$$5A = 7 - 6$$

$$A = 1/5$$

$$A + B = 2$$

$$\frac{1}{5} + B = 2$$

$$B = 2 - \frac{1}{5}$$

$$B = 9/5$$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{\frac{1}{5}}{(s-2)} + \frac{\frac{9}{5}s - 3}{(s^2 - 2s + 5)}$$

$$= \frac{1}{5}\frac{1}{s-2} + \frac{9}{5}\frac{s}{(s^2-2s+5)} - \frac{3}{(s^2-2s+5)}$$

$$= \frac{1}{5}\frac{1}{s-2} + \frac{9}{5}\left(\frac{s-1+1}{(s-1)^2+4}\right) - \frac{3}{2}\left(\frac{2}{(s-1)^2+4}\right)$$

$$L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{1}{5}\frac{1}{(s-2)} + \frac{9}{5}\left\{\frac{s-1}{(s-1)^2+2^2} + \frac{1}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right\} - \frac{3}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right\}$$

$$= L^{-1}\left\{\frac{1/5}{(s-2)} + \frac{9}{5}\left\{\frac{s-1}{(s-1)^2+2^2} + \frac{1}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right\} - \frac{3}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right\}$$

$$y = \frac{1}{5}e^{2t} + \frac{9}{5}\left\{e^t \cos 2t + \frac{1}{2}e^t \sin 2t\right\} - \frac{3}{2}(e^t \sin 2t)$$

3) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ given at $t=0$, $y=0$, y'

$$L\left\{\frac{d^2y}{dt^2}\right\} = s^2Y(s) - sy(0) - y'(0)$$

$$L\left\{-6\frac{dy}{dt}\right\} = -6sY(s) + 6y(0)$$

$$L\{8y\} = 8Y(s)$$

$$L\{e^{3t}\} = \frac{1}{s-3}$$

$$s^2Y(s) - sy(0) - y'(0) - 6sY(s) + 6y(0) + 8Y(s) = \frac{1}{s-3}$$

$$s^2Y(s) - 6sY(s) + 8Y(s) - 2 = \frac{1}{s-3}$$

$$Y(s)(s^2 - 6s + 8) = \frac{1}{s-3} + 2$$

$$Y(s)(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{s-3}$$

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$$Y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{s-3} + \frac{Bs+C}{(s^2-6s+8)}$$

$$2s-5 = A(s^2-6s+8) + (Bs+C)(s-3)$$

$$2s-5 = As^2 - 6As + 8A + Bs^2 - 3Bs + Cs - 3C$$

$A+B=0$

$-6A-3B+C=2$

$8A-3C=-3$

$B=-A$ from (1)

$-6A+3A+C=2$

$-3A+C=2$ ... (4)

$8A-3C=-5$ ... (5)

$9A-3C=-6$

$8A-3C=-5$

$A=-1$

$B=1$

from (4)

$C=2-3$

$C=-1$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{s-3} + \frac{s-1}{(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{s-3} + \frac{s-1}{(s-2)(s-4)}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$s-1 = A(s-4) + B(s-2)$

$s-1 = As - 4A + Bs - 2B$

$A+B=1$ ... (1) $\times -4$

$-4A-2B=-1$ ... (2) $\times 1$

$-4A-4B=-4$

$-4A-2B=-1$

$-2B=-3$

$B=\frac{3}{2}$

$A=-\frac{1}{2}$

$$\frac{s-1}{(s-2)(s-4)} = \frac{-\frac{1}{2}}{s-2} + \frac{\frac{3}{2}}{s-4}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{s-3} + \left(\frac{-\frac{1}{2}}{(s-2)} + \frac{\frac{3}{2}}{s-4}\right)$$

$$L^{-1}(Y(s)) = L^{-1}\left\{\frac{-1}{s-3} - \frac{\frac{1}{2}}{s-2} + \frac{\frac{3}{2}}{s-4}\right\}$$

$$y = -e^{3t} - \tfrac{1}{2}e^{2t} + \tfrac{3}{2}e^{4t}$$

$$y = \frac{1}{2}\left\{2e^{3t} + e^{2t} - 3e^{4t}\right\}$$

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