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15/ENG03/006
Civil Engineering

1) $(1-x^3)\frac{dy}{dx} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^3)y^2 - 2xy' + 2y = 0$

$y^n = U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \ldots$

$[y^{(2+n)} \cdot (1-x^2) + ny^{(1+n)} \cdot (-2x) + \frac{n(n-1)}{2!}y^n \cdot (-2)] + [y^{(x+n)} \cdot$

$+ny^n \cdot 2 + [2y^n] = 0$

$(1-x^2)y^{(2+n)} - 2xny^{(1+n)} - n(n-1)y^{(n)} - 2xy^{(1+n)} - 2ny^{(n)} +$

at $x = 0$

$y^{(2+n)} - n(n-1)y^{(n)} - 2ny^{(n)} + 2y^n = 0$

$y^{n+2} + y^n[-n(n+1) - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 - n + 2] = 0$

$n = 0 \therefore y^2 = -y^0 \cdot 2 = -2y^0$

$n = 1 \quad y^3 = -y' \cdot [0] = 0$

$n = 2 \quad y^4 = -y^2[-4] = 4y^2 = 4(-2y^0) = -8y^0$

$n = 3 \quad y^5 = -y^3[-10] = -10y^3 = 10 \cdot 0 = 0$

$n = 4 \quad y^6 = -y^4[-18] = 18y^4 = 18 \cdot 4 = -2y^n$

$n = 5 \quad y^7 = -y^5 \cdot [-28] = 28(y^3) = 28 \cdot 0 = 0$

$y = y^0 + xy' + \frac{x^2}{2!}y^3 + \frac{x^3}{3!}y^3 + \ldots$

$y = y^0 + xy' + \frac{x^2}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)y^2 + \ldots$

$+ \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)(-2)y^0 + \frac{x^7}{7!}(0)$

$y = y^0 + xy' - x^2y^0 - \frac{x^4}{3}y^0 - \frac{x^6}{5}y^0$

$y = y^0[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}] + y_0'[x]$

2) $3\frac{dy}{dt} - 6y = \sin 2t$

at $t=0, y=1$

$3(sy(s)) - y((0)) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+4}$

$3y(s)(s-2) - 3 = \frac{2}{s^2+4}$

$3y(s)(s-2) = \frac{2}{s^2+4} + 3$

$\frac{2 + 3s^2 + 12}{s^2+4}$

$y(s) = \frac{3s^2+14}{3(s-2)(s^2+4)}$

$= \frac{3s^2+14}{(3s-6)(s^2+4)}$

$y(s) = L^{-1}\left[\frac{3s^2+14}{(3s-6)(s^2+4)}\right]$

$\frac{A}{(3s-6)} + \frac{Bs+C}{(s^2+4)}$

$3s^2+14 = A(s^2+4) + Bs + C(3s-6)$

at $s=2$

$12+14 = A(8) + (Bs+C)(0)$

$\frac{26}{8} = \frac{8A}{8}$

$A = 13/4$

Using the method of coefficient

$3 = A + B$ — ①

$3 = 13/4 + 3B$

$3B = 3 \cdot \frac{13}{4}$

$3B = \frac{12-13}{4}$

$3B = -1/4$, $B = -1/12$

$14 = 4A - 6C$

$14 = 4 \times \frac{13}{4} - 6C$

$14 = 13 - 6C$

$1 = -6C$

$C = -1/6$

$y(s) = L^{-1}\left[\frac{13}{14} \times \frac{1}{3s-6} + \frac{(-1/12 s - 1/6)}{s^2+4}\right]$

$= L^{-1}\left[\frac{13}{12} \times \frac{1}{s-2} - \frac{1s}{12} \times \frac{1}{s^2+4} - \frac{1}{6} \times \frac{1}{s^2+4}\right]$

$= L^{-1}\left[\frac{13}{12} \times \frac{1}{s-2}\right] + L^{-1}\left[\frac{-1}{12} \times \frac{s}{s^2+4}\right] + L^{-1}\left[\frac{1}{6\times2} \times \frac{2}{s^2+4}\right]$

$= \frac{13}{12} e^{2t} - \frac{1}{12} \cos 2t + \frac{1}{12} \sin 2t$

3) $dy - 4y = 8$

at $t=0$, $y=2$

$3y(s) - y(0) - 4y(s) = \frac{8}{5}$

$5y(s) - 2 - 4y(s) = \frac{8}{5}$

$y(s)(s-4) = \frac{8}{5} + 2$

$y(s)(s-4) = \frac{8+2s}{5}$

$y(s) = \frac{8+2s}{s(s-4)}$

$y(s) = L^{-1} \frac{8+2s}{s(s-4)} = L^{-1}\left[\frac{A}{s} + \frac{B}{s-4}\right]$

$\frac{8+2s}{s(s-4)} = \frac{A(s-4) + B(s)}{s(s-4)}$

4) $\frac{d^2y}{dt^2}$ -

$8+2s$

$8+$

$y(s)$

s^2y

$y(s)$

$$8+2s = A(s-4)+B(s)$$
when $s=4$
$$8+8 = A(0)+4B$$
$$B=4$$
when $s=0$
$$8 = -4A+0$$
$$\frac{-4A}{4} + \frac{8}{4}$$
$$A=-2$$
$$y(s) = L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right]$$
$$y(s) = -2+4e^{4t}$$

4) $$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$$

$$s^2y(s) - 5y(0) - y'(0) - 2\left(sy(s) - y(0)\right) + 5y(s) = \frac{1}{s-2}$$

at $t=0$, $y=2$, $y'=1$
$$s^2y(s) - s(2) - 1 - 2(sy(s)-2) + 5y(s)$$
$$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s)$$
$$s^2y(s) - 2sy(s) + 5y(s) - 2s+3 = \frac{1}{s-2}$$

$$y(s)\,(s^2-2s+5) - 2s+3 = \frac{1}{s-2}$$

$$y(s)\,(s^2+2s+5) = \frac{1}{s-2} + \frac{2s}{1} + \frac{3}{1}$$

$$= \frac{1+2s^2-4+3s-6}{s-2}$$

$$y(s)\,s^2-2s+5 = \frac{2s^2+3s-9}{s-2}$$

$$y(s) = \frac{2s^2-13s-9}{(s-2)(s^2-2s+5)}$$

$$L^{-1}\left[\frac{2s^2+3s-9}{(s-2)(s^2-2s+5)}\right] = \left[\frac{A}{(s-2)} + \frac{Bs+C}{s^2-2s+5}\right]$$

$$2s^2+3s-9 = A(s^2-2s+5) + (Bs+C)(s-2)$$

at $s=2$

$8+6-9 = A(4-8+5)+0$

$A=5$

Using the method of coefficient

$2=A+B$

$2=5+B$

$B=-3$

$3=-2A-2B+C$

$3=-2(5)-2(-3)+C$

$3=-10+6+C$

$3=-4+C$

$C=-7$

$$y(s) = \left[\frac{5}{s-2} + \frac{(-3s-7)}{s^2-2s+5}\right]$$

$$= L^{-1}\left[\frac{5}{s-2}\right] - L^{-1}\left[\frac{3s}{(s-1)^2+4}\right]$$

$$= L^{-1}\left[\frac{7}{(s-1)^2+4}\right]$$

$$= L^{-1}\left[\frac{5}{s-2}\right] - L^{-1}\left[\frac{3\times 5}{(s-1)^2+4}\right]$$

$$= L^{-1}\left[\frac{7}{2} \times \frac{2}{(s-1)^2+4}\right]$$

$$= 5e^{2t} - 3e^{t}\cos 2t - \tfrac{7}{2}e^{t}\sin 2t$$

5) $\frac{d^2y}{dt^2} - \frac{6dy}{2t} + 8y = e^{3t}$

at $t=0$, $y=0$, $y'=2$

$= s^2y(s) - sy(0) - y'(0) - 6(sy(s) - y(s)) + 8y(s)$

$= s^2y(s) - 0 - 2 - 6sy(s) + 6(0) + 8y(s) = \cancel{\tfrac{1}{s}}\ \frac{1}{s-3}$

$= s^2y(s) - 2 - 6sy(s) + 8y(s) = \frac{1}{s-3}$

$$= y(s)\,(s^2-6s+8) = \frac{1}{s-3} + \frac{2}{1}$$

$$= y(s)\,(s^2-6s+8) = \frac{1+2s+6}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$y(s) = L^{-1}\left[\frac{2s-5}{(s-3)(s-4)(s-2)}\right]$$

$$\frac{2s-5}{(s-3)(s-4)(s-2)} = \frac{A}{s-3} + \frac{B}{s-4} + \frac{C}{s-2}$$

$$\frac{2s-5}{(s-3)(s-4)(s-2)} = \frac{A(s-4)(s-2) + B(s-3)(s-2) + C(s-3)(s-4)}{(s-3)(s-2)(s-4)}$$

$$2s-5 = A(s-4)(s-4) + B(s-3)(s-2) + C(s-3)(s-4)$$

at $s=2$

$$4-5 = A(0) + B(0) + C(-1)(-2)$$

$$-1 = 2C$$

$$C = -\tfrac{1}{2}$$

$$s=3$$

$$6-5 = A(-1)(1) + B(0) + C(0)$$

$$1 = -A \quad \therefore A = -1$$

at $s=4$

$$8-5 = A(0) + B(2)(1) + C(0)$$

$$3 = 2B$$

$$B = \tfrac{3}{2}$$

$$y(s) = L^{-1}\left[\frac{-1}{s-3} + \frac{3}{2} \times \frac{1}{s-4} - \frac{1}{2} \times \frac{1}{s-2}\right]$$

$$y(s) = -e^{3t} + \frac{3}{2}e^{4t} - \frac{1}{2}e^{2t}$$