$\cancel{L\{1\} = \frac{1}{s}}$ $\cancel{L\{e^{3t}\} = \frac{1}{s-3}}$

$\cancel{L\{\sin 2t\} = \frac{2}{s^2+4}}$

$\cancel{L\{\cos 2t\} = \frac{s}{s^2+4}}$

i) $3e^{-4t} - 5e^{4t}$

$L\{3e^{-4t} - 5e^{4t}\} = L\{3e^{-4t}\} - L\{5e^{4t}\}$

$= 3L\{e^{-4t}\} - 5L\{e^{4t}\}$

$= \frac{3}{s+4} - \frac{5}{s-4}$

$= \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)}$

$= \frac{3s - 12 - 5s - 20}{s^2 - 16^2}$

$= \frac{-2s - 32}{s^2 - 16^2}$

ii) $\sin 4t + \cos 4t = L\{\sin 4t\} + L\{\cos 4t\}$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

$= \frac{4+s}{s^2+16} = \frac{9}{s^2+16}$ //

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iii) $t^3 + 2t^2 - t + 4$

$= L\{t^3\} + 2L\{t^2\} - L\{t\} + L\{4\}$

$= \frac{3!}{s^4} + \frac{2 \cdot 2!}{s^3} - \frac{1!}{s^2} + \frac{1}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{1}{s}$

$= \frac{6 + 4s - s^2 + s^3}{s^4}$

$= \frac{s^3 - s^2 + 4s + 6}{s^4} = \frac{1}{s^4}\{s^3 - s^2 + 4s + 6\}$

$a = 2,\ w = 5$

iv) $e^{-2t}\cos 5t = L\{e^{-2t}\cos 5t\}$

$L\{\cos 5t\} = \frac{s}{s^2+25}$ $\therefore L\{e^{-2t}\cos 5t\} = \frac{s+2}{(s+2)^2+25}$

v) $t\sin 3t = L\{t\sin 3t\}$

$\Rightarrow L\{\sin 3t\} = \frac{3}{s^2+9}$

$L\{t\sin 3t\} = -\frac{d}{ds}\left(\frac{3}{s^2+9}\right) = \frac{6s}{(s^2+9)^2}$

vi) $\frac{e^{-t} - e^{-2t}}{t} = t^{-1}e^{-t} - t^{-1}e^{-2t}$

$f(t) = \frac{-1!}{(s+1)^{-1+1}} - \frac{1!}{(s+2)^{-1+2}}$

$f(s) = \frac{-1}{1} + \frac{1}{s+2}$

$f(s) = -1 - \frac{1}{s+2}$

$= \frac{-s-3}{s+2}$

vii) $e^{4t}\cos 2t = f(t)$

$L\{\cos 2t\} = \frac{s}{s^2+4}$

$f(s) = \frac{s-4}{(s-4)^2+4}$

viii) $t \sin 2t$

$L\{\sin 2t\} = \frac{2}{s^2+4}$

Recall $f(s) = -\frac{d^n}{ds^n}\left(\frac{2}{s^2+4}\right)$

$f(s) = -\left(\frac{s^2+4(0)-2(2s)}{(s^2+4)^2}\right) = \frac{4s}{(s^2+4)^2}$ //

ix) $t^3+4t^2+5 = f(t)$

$f(s) = L\{f(t)\}$

$f(s) = \frac{3!}{s^4} + 4\cdot\frac{2!}{s^3} + \frac{5}{s}$

$f(s) = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x) $e^{3t}(t^2+4) = f(t) = t^2e^{3t} + 4e^{3t}$

$f(s) = L\{f(t)\}$

$f(s) = \frac{2}{(s-3)^3} + \frac{4}{s-3}$

xi) $t^2\cos t$

$f(s) = (-1)^n\frac{d^n}{ds^n}$

$L\{\cos t\} = \frac{s}{s^2+1}$

$f(s) = (-1)^2\frac{d^2 s}{ds^2(s^2+1)}$

$= -1\left(\frac{s^2+(1) - s(2s)}{(s^2+1)^2}\right)$

$= \left(\frac{s^2+1 - 2s^2}{(s^2+1)^2}\right) = \frac{-1+s^2}{(s^2+1)^2}$

$= \frac{(s^2+1)^2(-2s) - (1-s^2)(4s^3+4s)}{(s^2+1)^4}$

$= \frac{(s^2+1)(2s) + (-s^2+1)(4s^3+4s)}{(s^2+1)^4}$

$= \frac{\cancel{(s^2+1)}(2s-4s^3+4s)}{\cancel{(s^2+1)}^4} = \frac{6s-4s^3}{(s^2+1)^3}$

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xii) $\frac{\sinh 2t}{t} = f(t)$

$f(s) = \int \frac{2s}{s^2-4}$ $\therefore f(s) = \ln(s^2-4)$

③ i) $\frac{s-5}{(s-3)(s-4)} = f(s)$

$\frac{A}{(s-4)} + \frac{B}{(s-3)} = \frac{s-5}{(s-4)(s-3)}$

$A|s=4 = \frac{4-5}{(4-3)} = \frac{-1}{1} = -1$

$B|s=3 = \frac{3-5}{3-4} = \frac{-2}{-1} = 2$

$\therefore A = -1$ and $B = 2$

$f(s) = \frac{-1}{(s-4)} + \frac{2}{(s-3)}$

$f(t) = \mathcal{L}^{-1}[F(s)]$

$= -e^{4t} + 2e^{3t}$

$= 2e^{3t} - e^{4t}$

ii) $\frac{2s-6}{(s-2)(s-4)}$

$\frac{A}{(s-2)} + \frac{B}{(s-4)} = \frac{2s-6}{(s-2)(s-4)}$

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$A|s=2 = \frac{2(2)-6}{(2-4)} = 1$

$B|s=4 = \frac{2(4)-6}{4-2} = 1$

$A = 1$ and $B = 1$

$f(s) = \frac{1}{s-2} + \frac{1}{s-4}$

$f(t) = \mathcal{L}^{-1}\{f(s)\}$

$f(t) = e^{2t} + e^{4t}$

iii) $\frac{5s-8}{s(s-4)} = f(s)$

$= \frac{A}{s} + \frac{B}{s-4} = \frac{5s-8}{s(s-4)}$

$A|s=0 = \frac{5(0)-8}{(0-4)} = 2$

$B|s=4 = \frac{5(4)-8}{4} = 3$

$A = 2$ and $B = 3$

$f(s) = \frac{2}{s} + \frac{3}{s-4}$

$f(t) = \mathcal{L}^{-1}\{F(s)\}$

$f(t) = 2 + 3e^{4t}$

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iv) $\dfrac{s^3 - 3s - 4}{(s-3)(s-1)^2} = f(s)$

$\dfrac{A}{s-3} + \dfrac{B}{(s-1)^2} + \dfrac{C}{(s-1)} = \dfrac{s^3 - 3s - 4}{(s-3)(s-1)^2}$

$A|s = 3 = \dfrac{3^3 - 3(3) - 4}{(3-1)^2}$

$= \dfrac{27 - 9 - 4}{4} = \dfrac{14}{4} = \dfrac{7}{2}$

$B|s = 1 = \dfrac{d}{ds}\left(\dfrac{s^3 - 3s - 4}{s-3}\right)$

$B|s = 1 = \dfrac{2s^3 - 9s^2 + 13}{(s-3)^2}$

$B|s = 1 = \dfrac{2 - 9 + 13}{4} = \dfrac{6}{4} = \dfrac{3}{2}$

$C|s = 1 = \dfrac{s^3 - 3s - 4}{s-3} = \dfrac{1^3 - 3(1) - 4}{1-3} = 3$

$A = \frac{7}{2}$, $B = \frac{3}{2}$ and $C = 3$

$F(s) = \dfrac{7}{2(s-3)} + \dfrac{3}{2(s-1)^2} + \dfrac{3}{(s-1)}$

$f(t) = \frac{7}{2}e^{3t} + \frac{3}{2}te^{t} + 3e^{t}$ //

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2x' + 2y^0 = 0$

Let $u^0 = y''$, $u' = y'''$, $u'' = y^{iv}$

$v^0 = (1-x^2)$ $\quad v' = -2x \quad v'' = -2$

$w_1 = y^{n+2}(1-x^2) + ny^{n+1}(-2x) + \dfrac{(-2)(n)(n-1)y^n}{2!}$

$w_2 = -(y^{n+1}(2x) + ny^n 2)$

$y^{n+2}(1-x^2) - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1}$

$- 2ny^n + 2y^n = 0$

$0 = y^{n+2}(1-x^2) + y^{n+1}(2xn - 2x) + y^n(-2n + 2 - n(n-1))$

when $x = 0$,

$y^{(n+2)}(1-0) + y^{n+1}(0-0) + y^n(-n^2 - n + 2) = 0$

$y^{n+2} + y^n(-n^2 - n + 2) = 0$

$y^{n+2} = y^n(n^2 + n - 2) = 0$

when $n = 0$ $\quad y_0^{0+2} = y_0^0(0+0-2) = y_0^0(-2) = -2y_0^0$

$n = 1 \quad y_0^3 = y_0^1(1^2+1-2) = y_0^1(0) = 0$

$n = 2 \quad y_0^4 = y_0^2(2^2+2-2) = y_0^2(4) = 4 \times 2y^0$

$n = 3 \quad y_0^5 = y_0^3(3^2+3-2) = y_0^3(10) = 10 \times 0$

$n = 4 \quad y_0^6 = y_0^4(4^2+4-2) = y_0^4(18) = 18 \times 4 \times -2y_0^0$

$n = 5 \quad y_0^7 = y_0^5(5^2+5-2) = y_0^5(28) = 28 \times 10 \times 0$

$n = 6 \quad y_0^8 = y_0^6(6^2+6-2) = y_0^6(40) = 40 \times 18 \times 4 \times -2y_0^0$

$y = y_0^0 + xy_0^1 + \dfrac{x^2}{2!}\cdot y_0^2 + \dfrac{x^3}{3!}\cdot y_0^3 + \dfrac{x^4}{4!}\cdot y_0^4 + \dfrac{x^5}{5!}\cdot y_0^5$

$+ \dfrac{x^6}{6!}\cdot y_0^6 + \dfrac{x^7}{7!}y_0^7 + \dfrac{x^8}{8!}y_0^8$

$$y = y_0^0 + x y_0' + \frac{x^2}{2\times 1}(-2y_0^0) + \frac{x^4}{4\times 3\times 2\times 1}(4\times -2y_0^0) + \frac{x^6}{6\times 5\times 4\times 3\times 2\times 1}(18\times 4\times -2\ldots$$

$$+ \frac{x^8}{8\times 7\times 6\times 5\times 4\times 3\times 2\times 1}(40\times 18\times 4\times -2y^0)$$

$$y = y_0^0\left(1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} - \frac{x^8}{7} + \cdots +\right) + x y_0'$$