

Name: - Mgbeze Emmanuel - c.
Mechatronics
15/EN605/013

$$1 \quad \frac{dy}{dt} + 3y = e^{-2t}$$

$$y(t) + 3y(t) = e^{-2t}$$

$$L[y(t)] = sY(s) - y(0)$$

$$L[e^{-2t}] = \frac{1}{s+2}$$

$$L^{-1}\left[\frac{1}{s+2}\right] = \frac{1}{s+2}$$

$$sY(s) - y(0) - 3Y(s) = \frac{1}{s+2}$$

$$(s-3)Y(s) - 2 = \frac{1}{s+2}$$

$$(s-3)Y(s) = \frac{1}{s+2} + 2 = \frac{1+2s+4}{s+2} = \frac{2s+5}{s+2}$$

$$Y(s) = \frac{2s+5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$$

$$2(-2) + 5 = A(-2-3) \Rightarrow A = -1/5$$

$$2(3) + 5 = 3(3+2) \Rightarrow B = 11/5$$

$$Y(s) = \frac{-1}{s+2} + \frac{11}{s-3} = -\frac{1}{5}e^{-2t} + \frac{11}{5}e^{3t}$$

$$(ii) \quad 3 \frac{dy}{dt} - 6y = \sin 2t = 3y(t) - (y(t))$$

$$L[y(t)] = sY(s) - y(0)$$

$$L[y(t)] = Y(s)$$

$$L[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$

$$3s + 7(s) - 37(s) - 67(s) = \frac{2}{s^2+4}$$

$$7(s) [3s-6] = \frac{2}{s^2+4} + 3 = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$$

$$7(s) = \frac{3s^2+14}{(3s-6)(s^2+4)} = \frac{A}{3s-6} + \frac{B}{s^2+4}$$

$$3s^2+14 = A(s^2+4) + B(3s-6)$$

$$4A - 6B = 14$$

$$B = -1/3$$

$$7(s) = \frac{3}{3(s-2)} - \frac{1}{3(s^2+4)}$$

$$= e^{2t} - \frac{1}{6} \sin 2t$$

(iii) $\frac{dy}{dx} - 4y = 8$

$$y'(t) - 4y(t) = 8$$

$$y'(t) - 4y(t) = 8$$

$$S y(s) - y(0) - 4y(s) = 8/s$$

$$(s-4)y(s) = 8/s + 2 = \frac{8+2s}{s}$$

$$y(s) = \frac{2s+8}{s(s-4)}$$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s+8 = A(s-4) + B(s)$$

$$2(6) + 8 = A(0-4)$$

$$2(4) + 8 = A(4-4) + B(4) = B = 4$$

$$L^{-1} \left[\frac{2}{s} + \frac{4}{s-4} \right] = -2 + 4e^{4t}$$

$$(w) \frac{d^2 y}{dt^2} - 2 \frac{dy}{dt} + 5y = e^{2t}$$

$$y''(t) - 2y'(t) + 5y(t) = e^{2t}$$

$$L[y''(t)] = s^2 Y(s) - sy(0) - y'(0)$$

$$L[y'(t)] = sY(s) - y(0)$$

$$L[y(t)] = Y(s)$$

$$s^2 Y(s) - sy(0) - y'(0) - 2(sY(s) - y(0)) + 5Y(s) = \frac{1}{s-2}$$

$$(s^2 - 2s + 5)Y(s) + (2-5)y_0 - y'(0) = \frac{1}{s-2}$$

$$(s^2 - 2s + 5)Y(s) = \frac{1}{s-2} + 2s - 4 + 1$$

$$= \frac{1}{s-2} + 2s - 3 = \frac{1 + (2s-3)(s-2)}{(s-2)}$$

$$= \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$$

$$Y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 3s + 5)}$$

$$(s-2)(s^2 - 3s + 5)$$

$$2s^2 = 7s + 7 = \frac{A}{s-2} + \frac{B}{s^2 - 3s + 5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s - 2)$$

$$A = 2$$

$$2A + B = -7$$

$$B = -3$$

$$Y(s) = \frac{2}{s^2} - \frac{3}{s-2s+5}$$
$$= 3e^{2t} = \frac{1}{3} + \sin 2t$$