

15/ENGG081025

Ogboju. P. Ofiri

ENGG 881

Assignment 5

$$i) \frac{dy}{dt} + 8y = e^{-2t}$$

$$y(t) + 8y(t) = e^{-2t}$$

$$L[y'(t)] = 8Y(s) - Y(s)$$

$$L[y(t)] = Y(s), L^{-1}[e^{-2t}] = \frac{1}{s+2}$$

$$8Y(s) - Y(s) + 8Y(s) = \frac{1}{s+2}$$

$$(8-8)Y(s) - 2 = \frac{1}{s+2}$$

$$(8-8)Y(s) = \frac{1}{s+2} + 2 = \frac{1+2s+4}{s+2} = \frac{2s+5}{s+2}$$

$$Y(s) = \frac{2s+5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$$

$$(s+2)(s-3) \quad s+2 \quad s-3$$

$$2(-2) + 5 = A(-2-3) \Rightarrow A = -\frac{1}{5}$$

$$2(3) + 5 = B(3+2) \Rightarrow B = \frac{11}{5}$$

$$Y(s) = \frac{-1}{5(s+2)} + \frac{11}{5(s-3)} = \frac{-1}{5} e^{-2t} + \frac{11}{5} e^{3t}$$

$$ii) \frac{dy}{dt} - 6y = 2 \sin t = 8y'(t) - 6y(t)$$

$$L[y'(t)] = 8Y(s) - Y(s)$$

$$L[y(t)] = Y(s)$$

$$L[2 \sin t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$

$$8Y(s) - 6Y(s) - 6Y(s) = \frac{2}{s^2+4}$$

$$Y(s) [8-6-6] = \frac{2}{s^2+4} + 6 = \frac{2+6s^2+12}{s^2+4} = \frac{6s^2+14}{s^2+4}$$

$$Y(s) = \frac{6s^2+14}{s^2+4} = \frac{A}{s-6} + \frac{B}{s^2+4}$$

$$(6-6)(s^2+4) \quad 6s-6 \quad s^2+4$$

$$6s^2+14 = A(s^2+4) + B(6s-6)$$

$$4A - 6B = 14$$

$$-6B = 14 - 12$$

$$B = \frac{-1}{3}$$

$$Y(s) = \frac{8}{[s][s^2]} - \frac{1}{s(s^2+4)}$$

$$= e^{2t} - \frac{1}{6} \sin 2t$$

iii) $\frac{dy}{dt} - 4y = 8$

$$Y'(t) - 4Y(s) = 8$$

$$Y'(t) - 4y(t) = 8$$

$$sY(s) - Y(0) - 4Y(s) = \frac{8}{s}$$

$$(s-4)Y(s) = \frac{8}{s} + 2 = \frac{8+2s}{s}$$

$$Y(s) = \frac{2s+8}{s(s-4)}$$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s+8 = A(s-4) + B(s)$$

$$2(0)+8 = A(0-4) \Rightarrow A = -2$$

$$2(4)+8 = A(4-4) + B(4)$$

$$B = 4$$

$$L^{-1} \left[\frac{-2}{s} + \frac{4}{s-4} \right] = -2 + 4e^{4t}$$

iv) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$$Y''(t) - 2Y'(t) + 5Y(t) = e^{2t}$$

$$L[Y''(t)] = s^2 Y(s) - sY(0) - Y'(0)$$

$$L[Y'(t)] = sY(s) - Y(0)$$

$$L[Y(t)] = Y(s)$$

$$s^2 Y(s) - sY(0) - Y'(0) - 2[sY(s) - Y(0)] + 5Y(s) = \frac{1}{s-2}$$

$$[s^2 - 2s + 5] Y(s) + (2-s)Y(0) - Y'(0) = \frac{1}{s-2}$$

$$(s^2 - 2s + 5) Y(s) = \frac{1}{s-2} - (2-s)Y(0) + Y'(0)$$

$$= \frac{1}{s-2} + 2s - 4 + 1 = \frac{1}{s-2} + 2s - 3$$

$$= \frac{1 + (2s-3)(s-2)}{(s-2)} = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$$

$$Y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$$

$$2s^2 - 7s + 7 = \frac{A}{s-2} + \frac{B}{s^2 - 2s + 5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$$

$$A = 2$$

$$-2A + B = -7$$

$$B = 2A - 7$$

$$B = 2(2) - 7 = 4 - 7 = -3$$

$$= \frac{2}{s-2} - \frac{3}{s^2 - 2s + 5}$$

$$= 2e^{2t} - \frac{7}{8} \sin 2t$$

$$v) \frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = e^{3t}$$

$$s^2 Y(s) - 3Y(s) - Y'(s) - (6sY(s) + 6Y(s) + 8Y(s)) = \frac{1}{s-3}$$

$$[s^2 - 6s + 8] Y(s) = \frac{1}{s-3} + 2 = \frac{1+2s-6}{s-3} = \frac{2s-5}{s-3}$$

$$Y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$2s-5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-3)(s-2)$$

$$2s-5 = A(s^2 - 6s + 8) + B(s^2 - 7s + 12) + C(s^2 - 5s + 6)$$

$$2(8) - 5 = A(3-2)(3-4) \Rightarrow A = -1$$

$$2(4) - 5 = C(4-3)(4-2) \Rightarrow C = \frac{3}{2}$$

$$-6A - 7B - 5C = 7$$

$$-6[-1] - 7(B) - 5(\frac{3}{2}) = 7$$

$$-7B = 7 + \frac{15}{2} - 6 = 4 + \frac{15-12}{2} = \frac{7}{2} \Rightarrow B = -\frac{1}{2}$$

$$2^{-1} \left[\frac{1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)} \right]$$

$$= -e^{3t} - \frac{1}{2} e^{2t} + \frac{3}{2} e^{4t}$$