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15 King 02/035
Comp. Eng.
Eng 381 Assignment

$$i) \frac{dy}{dt} + 3y = e^{-3t}$$

$$y(t) + 3y(t) = e^{-3t}$$

$$L[y'(t)] = Y(s), \quad L[e^{-3t}] = \frac{1}{s+3}$$

$$sY(s) - Y(0) + 3Y(s) = \frac{1}{s+3}$$

$$sY(s) - Y(0) + 3Y(s) = \frac{1}{s+3}$$

$$(s-3)Y(s) - 2 = \frac{1}{s+3}$$

$$(s-3)Y(s) = \frac{1}{s+3} + 2 = \frac{1+2s+6}{s+3} = \frac{2s+7}{s+3}$$

$$Y(s) = \frac{2s+7}{(s+3)(s-3)} = \frac{A}{s+3} + \frac{B}{s-3}$$

$$2(-2)+7 = A(-2-3) \Rightarrow A = \frac{1}{-5}$$

$$2(3)+7 = B(3+3) \Rightarrow B = \frac{11}{6}$$

$$Y(s) = \frac{-1}{5(s+3)} + \frac{11}{6(s-3)} = \frac{-1}{5} e^{-3t} + \frac{11}{6} e^{3t}$$

$$ii) 3 \frac{dy}{dt} - 6y = \sin 2t = 3y'(t) - 6y(t)$$

$$L[y'(t)] = sY(s) - Y(0)$$

$$L[y(t)] = Y(s)$$

$$L[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$

$$3s + Y(s) - 3Y(0) - 6Y(s) = \frac{2}{s^2+4}$$

$$Y(s) = \frac{3s^2+14}{(3s-6)(s^2+4)} = \frac{A}{3s-6} + \frac{B}{s^2+4}$$

$$4A - 6B = 14$$

$$-6B = 14 - 12$$

$$B = -1/3$$

$$\begin{aligned} Y(s) &= \frac{3}{[s][s-2]} - \frac{1}{3(s^2+4)} \\ &= e^{2t} - \frac{1}{6} \sin 2t \end{aligned}$$

$$(ii) \frac{dy}{dt} - 4y = 8$$

$$Y'(t) - 4Y(t) = 8$$

$$Y'(t) - 4y(t) = 8$$

$$sY(s) - Y(0) - 4Y(s) = \frac{8}{s}$$

$$(s-4)Y(s) = \frac{8}{s} + 2 = \frac{8+2s}{s}$$

$$Y(s) = \frac{2s+8}{s(s-4)}$$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s+8 = A(s-4) + B(s)$$

$$2(0)+8 = A(0-4) \Rightarrow A = -2$$

$$2(4)+8 = A(4-4) + B(4)$$

$$B = 4$$

$$L^{-1} \left[\frac{-2}{s} + \frac{4}{s-4} \right] = -2 + 4e^{4t}$$

$$(iv) \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$$

$$Y''(t) - 2Y'(t) + 5Y(t) = e^{2t}$$

$$L[Y''] - L[2Y'] + L[5Y] = L[e^{2t}]$$

$$L[Y'(t)] = sY(s) - Y(0)$$

$$L[Y(t)] = Y(s)$$

$$s^2 Y(s) - s Y(0) - Y'(0) - 2s Y(s) + 2 Y(s) + 5 Y(s) = \frac{1}{s-2}$$

$$[s^2 - 2s + 5] Y(s) + (2-s) Y(0) - Y'(0) = \frac{1}{s-2}$$

$$[s^2 - 2s + 5] Y(s) + (2-s) Y(0) - Y'(0) = \frac{1}{s-2}$$

$$[s^2 - 2s + 5] Y(s) = \frac{1}{s-2} - (2-s) Y(0) + Y'(0)$$

$$= \frac{1}{s-2} + 2s - 4 + 1 = \frac{1}{s-2} + 2s - 3$$

$$= \frac{1 + (2s-3)(s-2)}{(s-2)} = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$$

$$Y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$$

$$2s^2 - 7s + 7 = \frac{A}{s-2} + \frac{B}{s^2 - 2s + 5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$$

$$A = 2$$

$$-2A + B = -7$$

$$B = -7 + 4 = -3$$

$$= \frac{2}{s-2} - \frac{3}{s^2 - 2s + 5}$$

$$= 2e^{2t} - \frac{1}{3} t \sin 2t$$

$$(v) \frac{d^2 y}{dt^2} - 6 \frac{dy}{dt} + 8y = e^{3t}$$

$$s^2 Y(s) - s Y(0) - Y'(0) - 6s Y(s) + 6 Y(s) + 8 Y(s) = \frac{1}{s-3}$$

$$[s^2 - 6s + 8] Y(s) + (6-s) Y(0) - Y'(0) = \frac{1}{s-3}$$

$$[s^2 - 6s + 8] Y(s) = \frac{1}{s-3} + 2 = \frac{1 + 2s - 6}{s-3} = \frac{2s-5}{s-3}$$

$$Y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$2s-5 = A(s-2)(s-4) + B(s-3)(s-4) + [(s-3)(s-2)]$$

$$2s-5 = A(s^2 - 6s + 8) + B(s^2 - 7s + 12) + C(s^2 - 5s + 6)$$

$$2(3)-5 \neq 1 \Rightarrow A(3-2)$$

$$2(3)-5 = A(3-2)(3-4) \quad A \neq 1 \Rightarrow A = -1$$

$$2(4)-5 = C(4-3)(4-2) \Rightarrow C = 3/2$$

$$-6A - 7B - 5C = 7$$

$$-6[-1] - 7(B) - 5(3/2) = 7$$

$$-7B = 7 + \frac{15}{2} - 6 = \frac{4+15-12}{2} = \frac{7}{2} \Rightarrow B = -\frac{1}{2}$$

$$L^{-1} \left[\frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)} \right]$$

$$= -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$