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ENG381

① $dy/dt + 3y = e^{-2t}$

$y'(t) + 3y(t) = e^{-2t}$

$L[y'(t)] = sY(s) - y(0)$

$L[y(t)] = Y(s)$; $L^{-1}[e^{-at}] = \frac{1}{s+a}$

$sY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$

$(s-3)Y(s) - 2 = \frac{1}{s+2}$

$(s-3)Y(s) - 2 = \frac{1}{s+2}$

$(s-3)Y(s) = \frac{1}{s+2} + 2 = \frac{1+2s+4}{s+2} = \frac{2s+5}{s+2}$

$Y(s) = \frac{2s+5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$

$2(-2) + 5 = A(-2-3) \Rightarrow A = \frac{1}{-5}$

$2(3) + 5 = B(3+2) \Rightarrow B = 11/5$

$Y(s) = \frac{-1}{5(s+2)} + \frac{11}{5(s-3)} = \frac{-1}{5}e^{-2t} + \frac{11}{5}e^{3t}$

② $3\,dy/dx - 6y = \sin 2t$ $= 3y'(t) - 6y(t)$

$L[y'(t)] = sY(s) - y(0)$

$L[y(t)] = Y(s)$

$L[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$

$3sY(s) - 3y(0) - 6Y(s) = \frac{2}{s^2+4}$

$Y(s)[3s-6] = \frac{2}{s^2+4} + 3 = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$

$Y(s) = \frac{3s^2+14}{(3s-6)(s^2+4)}$

$= \frac{A}{3s-6} + \frac{B}{s^2+4}$

$3s^2+4 = A(s^2+4) + B(3s-6)$

$4A - 6B = 4$

$-6B = 14 - 12$

$B = -1/3$

$Y(s) = \frac{3}{[s][s-2]} - \frac{1}{3(s^2+4)}$

$= e^{2t} - \frac{1}{6}\sin 2t$

(i) $dy/dt - 4y = 8$

$y'(t) - 4y(t) = 8$

$y'(t) - 4y(t) = 8$

$sY(s) - Y(0) - 4Y(s) = 8/s$

$(s-4)[Y(s)] = 8/s + 2 = \frac{8+2s}{s}$

$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$2s+8 = A(s-4) + B(s)$

$2(0) + 8 = A(0-4) \Rightarrow A = -2$

$2(4) + 8 = A(4-4) + B(4)$

$B = 4$

$L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right] = -2 + 4e^{4t}$

(iv) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$y''(t) - 2y'(t) + 5y(t) = e^{2t}$

$L[y'(t)] = sY(s) - y(0) - Y(0)$

$L[y'(t)] = sY(s) - Y(0)$

$L[Y(0)] = Y(0)$

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$(s^2 - 2s+5)Y(s) + (2-s)6 - Y(s) = \frac{1}{s-2}$

$(s^2 + 2s + 5)Y(s) = \frac{1}{s-2} - [2-s]2+1$

$= \frac{1}{s-2} + 2s-4+1 = \frac{1}{s-2} + 2s-3$

$= \frac{1 + (2s-3)(s-2)}{(s-2)} = \frac{1+2s^2-4s-3s+6}{s-2}$

$Y(s) = \frac{2s^2-7s+7}{(s-2)(s^2-2s+5)}$

$2s^2 - 7s+7 = \frac{A}{s-2} + \frac{B}{s^2-2s+5}$

$2s^2 - 7s+7 = A(s^2 - 2s+5] + B(s-2)$

$A = 2$

$-2A+6 = -7$

$B = -7+4 = -3$

$= \frac{2}{s-2} - \frac{3}{s-2s+5}$

$= 2e^{2t} - 7/3 + \sin 2t$

v) $d^2y/dt^2 - 6dy/dx + 5y = e^3$

$s^2Y(s) - sy(0) - y'(0) - 6sY(s) + 6y(s) + 5Y(s) = \frac{1}{s-3}$

$[s^2-6s+8]Y(s) + (6-s)Y(0) - Y(0) = \frac{1}{s-3}$

$[s^2-6s+8]Y(s) = \frac{1}{s-3} + 2 = \frac{1+2s-6}{s-3} = \frac{2s-5}{s-3}$

$Y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$

$2s-5 = A(s-2)(s-4) + B(s-3)(s-4) + [(s-3)(s-2)]$

$2s-5 = A[s^2-6s+8] + B[s^2-7s+12] + C[s^2-5s+6]$

$2[3]-5 = A[3-2][3-4] \Rightarrow A = 1$

$2[4]-5 = C[4-3][4-2] \Rightarrow C = 3/2$

$-6[-1] - 7B - 5[1/2] = 2$

$-7B = 2 + 15/2 - 6 = \frac{4+15-12}{2} = 7/2$

$L^{-1}\left[\frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)}\right]^2 = -e^{3t} - 1/2\,e^{2t} + 3/2\,e^{4t}$ //

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