Name: Elendu Chidera Isabella
15/ENG02/021
Computer Engineering
ENG 381

1) $dy/dx + 3y = e^{-2x}$, given that at $t=0$, $y=2$

answer

$y'(x) + 3y(x) = e^{-2x}$

$\Rightarrow L\{y'(x)\} = sY_{(s)} - Y_{(0)}$

$\Rightarrow L\{y(x)\} = Y(s)$

$\Rightarrow L\{e^{-2x}\} = \frac{1}{s+2}$

$\Rightarrow$ substituting

$\Rightarrow sY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$

$\Rightarrow$ at $y=2$

$\Rightarrow sY(s) - 2 + 3Y(s) = \frac{1}{s+2}$

$\Rightarrow Y(s)[s+3] - 2 = \frac{1}{s+2}$

$\Rightarrow Y(s)[s+3] = \frac{1}{s+2} + 2$

$\Rightarrow Y(s)[s+3] = \frac{1+2(s+2)}{(s+2)}$

$\Rightarrow Y(s)[s+3] = \frac{1+2s+4}{s+2}$

$\Rightarrow Y(s)[s+3] = \frac{2s+5}{s+2}$

$\Rightarrow Y(s) = \frac{2s+5}{(s+2)(s+3)}$

$\Rightarrow Y(t) = L^{-1}\left[\frac{2s+5}{(s+2)(s+3)}\right]$

$\Rightarrow \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$\Rightarrow 2s+5 = A(s+3) + B(s+2)$

$\Rightarrow$ Let $s = -3$

$\Rightarrow 2(-3) + 5 = A\cancel{(-3+3)}^{0} + B(-3+2)$

$\Rightarrow -6 + 5 = -B$

$\Rightarrow -1 = -B$

$B = 1_{//}$

$\Rightarrow$ Let $s = -2$

$2(-2) + 5 = A(-2+3) + \cancel{B(-2+2)}^{0}$

$\Rightarrow -4 + 5 = 1A$

$\Rightarrow 1 = 1A$

$A = 1_{//}$

$\therefore y(t) = L^{-1}\left[\frac{1}{s+2} + \frac{1}{s+3}\right]$

$Y(t) \Rightarrow e^{-2t} + e^{-3t}$

) $3\,dy/dt - 6y = \sin 2t$, given that at $t = 0$, $y = 1$

answer

$\Rightarrow 3y'(x) - 6y(x) = \sin 2t$

$\Rightarrow L\{y'(x)\} = sY(s) - Y(0)$

$\Rightarrow L\{y(x)\} = Y(s)$

$\Rightarrow L\{\sin 2t\} = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$

Substituting,

$3\left[sY(s) - Y(0)\right] - 6Y(s) = \frac{2}{s^2+4}$

$\Rightarrow 3sY(s) - 3Y(0) - 6Y(s) = \frac{2}{s^2+4}$

$\Rightarrow Y(s)[3s-6] - 3Y(0) = \frac{2}{s^2+4}$

at $t=0$, $y=1$

$\Rightarrow Y(s)[3s-6] - 3(1) = \frac{2}{s^2+4}$

$\Rightarrow Y(s)\left[3s-6\right] = \dfrac{2}{s^2+4} + 3$

$\Rightarrow Y(s)\left[3s-6\right] = \dfrac{2+3(s^2+4)}{(s^2+4)}$

$\Rightarrow Y(s)\left[3s-6\right] = \dfrac{2+3s^2+12}{(s^2+4)}$

$\Rightarrow Y(s)\left[3s-6\right] = \dfrac{3s^2+14}{(s^2+4)}$

$\Rightarrow Y(s) = \dfrac{3s^2+14}{(s^2+4)(3s-3)}$

$\Rightarrow Y(t) = L^{-1}\left[\dfrac{3s^2+14}{(s^2+4)(3s-6)}\right]$

$\Rightarrow \dfrac{3s^2+14}{(s^2+4)(3s-6)} = \dfrac{A}{(3s-6)} + \dfrac{Bs+C}{(s^2+4)}$

$\Rightarrow 3s^2+14 = A(s^2+4) + (Bs+C)(3s-6)$

$\Rightarrow 3s^2+14 = A(s^2+4) + (Bs+C)(3s-6)$

Let $s = 2$

$\Rightarrow 12+4 = A(2^2+4) + (B(2)+C)(3(2)-6)$

$\Rightarrow 12+14 = A(8) + \cancel{(B4+C)}(6-6)$

$\Rightarrow 12+14 = 8A$

$\Rightarrow 12+14 = 8A$

$\Rightarrow 26 = 8A$

$\Rightarrow 26/8 = A$

$\therefore A = 13/4$

$\Rightarrow 3s^2+14 = As^2 + 4A + 3Bs^2 - 6Bs + 3Cs - 6C$

$\Rightarrow 3 = A + 3B$

$\Rightarrow 3 = 13/4 + 3B$

$\Rightarrow 3 - \dfrac{13}{4} = 3B$

$\Rightarrow \dfrac{12-13}{4} = 3B$

$\Rightarrow \dfrac{-1}{4} = 3B$

$\Rightarrow \dfrac{-1}{4} \times \dfrac{1}{3} = B$

$\Rightarrow B = -1/12$

$\Rightarrow 14 = 4A - 6c$

$\Rightarrow 14 = \cancel{4}(13/4) - 6c$

$\Rightarrow 14 = 13 - 6c$

$\Rightarrow 14 - 13 = -6c$

$\Rightarrow 1 = -6c$

$\Rightarrow -1/6 = c$

$$y(t) = L^{-1}\left[\frac{13/4}{(3s-6)} + \frac{\left(-\frac{1}{12}\right)s - 1/6}{(s^2+4)}\right]$$

$$\Rightarrow L^{-1}\left[\frac{13}{4}\frac{1}{3(s-2)} - \frac{1}{12}\frac{s}{s^2+4} - \frac{1}{6}\frac{1}{s^2+4}\right]$$

$$y(t) = \cancel{\frac{13}{12}}\left[\frac{1}{12} L^{-1}\left[\frac{13}{(s-2)} - \frac{s}{(s^2+4)} - \frac{2}{(s^2+4)}\right]\right]$$

$$y(t) = \frac{1}{12} 13e^{2t} - \cos 2t - \sin 2t$$

3) $\frac{dy}{dt} - 4y = 8$

at $t = 0$, $y = 2$

solution

$y'(x) - 4y(x) = 8$

$L[y'(x)] = sY(s) - Y(0)$

$L[y(x)] = Y(s)$

$L[8] = 8/s$

$\Rightarrow sY(s) - Y(0) - Y(s) = 8/s$

$\Rightarrow Y(s)[s-1] - Y(0) = 8/s$

$\Rightarrow Y(s)[s-1] - 2 = 8/s$

$\Rightarrow Y(s)[s-1] = 8/s + 2$

$\Rightarrow Y(s)[s-1] = \frac{8 + 2(s)}{s}$

$\Rightarrow Y(s) = \frac{8 + 2(s)}{s} \cdot \frac{1}{(s-1)}$

$\Rightarrow Y(s) = \frac{8 + 2(s)}{s(s-1)}$

$$\frac{8 + 2(s)}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1}$$

$2s+8 = A(s-4) + Bs$

Let $s = 0$

$2(0)+8 = A(0-4) + B(0)$

$0+8 = -4A + 0$

$8 = -4A$

$A = -2$

Let $s = 4$

$2(4)+8 = A\cancel{(4-4)}^{0} + B(4)$

$8+8 = 4B$

$16 = 4B$

$B = 4$ //

$y(t) = L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right]$

$y(t) = -2 + 4e^{4t}$ //

$= 4e^{4t} - 2$ //

4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ at $t=0$, $y = 2$, $y' = 1$

answer

$s^2 y(s) - s y(0) - y'(0) - 2[s y(s) - y(0)] + 5y(s) = \frac{1}{s-2}$

at

$s^2 y(s) - s(2) - 1 - 2[sy(s) - 2] + 5y(s) = \frac{1}{s+2}$

$s^2 y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s+2}$

$y(s)[s^2 - 2s + 5] - 2s - 1 + 4 = \frac{1}{s+2}$

$y(s)[s^2 - 2s + 5] = \frac{1}{s-2} + 2s - 3$

$Y(s)[s^2 - 2s + 5] = \frac{1 + 2s(s-2) - 3(s-2)}{s-2}$

$y(s)[s^2 - 2s + 5] = \frac{1 + 2s^2 - 4s - 3s - 6}{s-2}$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)}$$

$$y(t) = L^{-1}\left[\frac{A}{(s-2)} + \frac{Bs+C}{(s^2-2s+5)}\right]$$

$2s^2 - 7s + 7 = A(s^2-2s+5) + Bs + C\,(s-2)$

Let $s = 2$

$2(2)^2 - 7(2) + 7 = A(2^2 - 2(2) + 5) + \cancel{B(2) + C(2-2)} \to 0$

$8 - 14 + 7 = 5A$

$A = 1/5$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$5A - 2C = 7$ — (1)

$5(1/5) - 2C = 7$

$-2C = 7 + 1$

$-2C = 8$

$C = -4$ //

$-2A - 2B + C = -7$

$-2(1/5) - 2B - 4 = -7$

$-2/5 - 2B - 4 = -7$

$-2B = -7 + 4 + \frac{2}{5}$

$-2B = \frac{-35 + 20 + 2}{5}$

$-2B = -13/5$

$B = -13/5 \times -1/2$

$B = 13/10$

$$y(t) = L^{-1}\left[1/5 \; 1/s-2 + \frac{13/10\, s - -4}{s^2-2s+5}\right]$$

$$= L^{-1}\left[\frac{1}{5}\,\frac{1}{s-2} + \frac{13}{10}\,\frac{1}{s^2-2s+5} - \frac{4}{s^2-2s+5}\right]$$

$y(t) = \frac{1}{5}e^{2t} + \frac{13}{10}\,\frac{\sin 2t\, e^{t}}{2} - 4\,\frac{e^{t}\sin 2t}{2}$

$= \frac{1}{5}e^{2t} = \frac{13}{20}\sin 2t\, e^{t} - 2e^{t}\sin 2t$

= using L.C.M [Taking LCM]

$= 4e^{2t} + 13e^{t}\sin 2t - 40e^{t}\sin 2t$ //

5) $\frac{d^2y}{dx^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0$, $y=0$ and $y'=2$

answer

$y''(x) - 6y'(x) + 8y(x) = e^{3t}$

$\Rightarrow s^2y(s) - sy(0) - y'(0) - 6[sy(s) - y(0)] + 8y(s) = \frac{1}{s-3}$

at $t=0$, $y(0)=0$ and $y'=2$ //

$s^2y(s) - 0 - 2 - 6sy(s) + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - 6sy(s) + 8y(s) = \frac{1}{s-3} + 2$

$y(s)\,[s^2 - 6s + 8] = \frac{1+2(s-3)}{s-3}$

$y(s) = \frac{1+2s-6}{(s-3)(s^2-6s+8)}$

$y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)}$

$\frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$

$2s-5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-3)(s-2)$

let $s = 2$

$2(2)-5 = A\cancel{(2-2)}(2-4) + B(2-3)(2-4) + C\cancel{(2-3)}(2-2)$

$4-5 = B(-1)(-2)$

$-1 = 2B$, $B = -1/2$ //