OKOLIE CHINEDU
15/ENG06/052
MECHANICAL ENGINEERING

ENG 381

1 $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Let $8 = 0$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 - m - 2 = 0$

$m^2 + m - 2m - 2 = 0$

$m(m+1) - 2(m+1) = 0$

$(m-2)(m+1) = 0$

$m_1 = 2 \quad m_2 = -1$

$y = Ae^{2x} + Be^{-x}$

P.I : $y = c$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$0 - 0 - 2c = 8$

$-2c = 8$

$c = -4$

P.I $= -4$

G.S $=$ C.F + P.I

$= Ae^{2x} + Be^{-x} - 4$

2 $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Let $10e^{3x} = 0$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2 \quad m_1 = 2 \quad m_2 = -2$

$y = Ae^{2x} + Be^{-2x}$

P.I : $y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

divide through by e^{3x}

$9C - 4C = 10$

$5C = 10$

$C = 2$

P.I $= 2$

G.S $=$ C.F + P.I

G.S $= Ae^{2x} + Be^{-2x} + 2e^{3x}$

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

Let $C = e^{-2x}$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1) = 0$ twice

$m = -1$ twice

C.F : $y = e^{-x}(A + Bx)$

P.I $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

divide throughout by e^{-2x}

$4C - 4C + C = 1$

$C = 1$

G.S = C.F + P.I

G.S = $e^{-x}(A + Bx) + e^{-2x}$

$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

let $5x^2 + x = 0$

$m^2 + 25 = 0$

$m = \pm\sqrt{-25}$

$m = \pm 5$

C.F : $y = A\cos 5x + B\sin 5x$

P.I : $y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

* $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25C = 5$ --- (i)

$25D = 1$ --- (ii)

$2C + 25E = 0$ --- (iii)

from eqn (i)

$C = \frac{5}{25} = \frac{1}{5}$

from eqn (ii)

$D = \frac{1}{25}$

from eqn (iii)

$2\left(\frac{1}{5}\right) + 25E = 0$

$\frac{2}{5} = -25E$

$\frac{0.4}{-25} = E$

$E = -\frac{2}{125}$

P.I : $y = Cx^2 + Dx + E$

$y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

G.S = C.F + P.I

$= A\cos 5x + B\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

5 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

Let $4\sin x = 0$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$m - 1 = 0$ twice

$m = 1$ twice

C.F : $y = e^x(A + Bx)$

$F(x) = 4\sin x$

P.I : $y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$+2C\sin x - 2D\cos x = 4\sin x$

$2C = 4 \quad \cdots (i)$

$-2D = 0 \quad \cdots (ii)$

$D = 0$

$C = \frac{4}{2} = 2$

P.I : $y = C\cos x + D\sin x$

$= 2\cos x + 0\sin x$

$= 2\cos x$

G.S = C.F + P.I

$= e^x(A + Bx) + 2\cos x$

6 $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

Given that at $x = 0$, $y = 1$ and $\frac{dy}{dx} = -2$

Let $2e^{-2x} = 0$

$m^2 + 4m + 5 = 0$

$a = 1 \quad b = 4 \quad C = 5$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1}$

$m = \frac{-4 \pm \sqrt{-4}}{2 \times 1}$

$m = \frac{-4 \pm j2}{2 \times 1}$

$m = -2 \pm j \quad \alpha = -2 \quad \beta = 1$

C.F : $y = e^{-2x}(A\cos x + B\sin x)$

P.I : $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

divide through by e^{-2x}

$4C - 8C + 5C = 2$

$C = 2$

P.I $y = 2e^{-2x}$

G.S = C.F + P.I

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

7. $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

Let $2x - 3 = 0$

$3m^2 - 2m - 1 = 0$

$3m^2 - 3m + m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(m-1)(3m+1) = 0$

$m_1 = 1 \quad 3m_2 = -1$

$m = -\frac{1}{3}$

C.F ; $y = Ae^x + Be^{-x/3}$

$f(x) = 2x - 3$

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$3(2C) - 2(2Cx + D) - (Cx^2 + Dx + E) = 2x - 3$

$6C - 4Cx - 2D - Cx^2 - Dx - E = 2x - 3$

$-Cx^2 = 0$

$C = 0$

$-4C - D = 2$

$-4(0) - D = 2$

$-D = 2$

$D = -2$

$6C - E - 2D = -3$

$6(0) - E - 2(-2) = -3$

$-E + 4 = -3$

$-E = -7$

$E = 7$

$y = 0x^2 - 2x + 7$

$y = -2x + 7$

$y = Ae^x + Be^{-x/3} - 2x + 7$

8 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Let $8e^{4x} = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 =$

$m(m-4) - 2(m-4) =$

$(m-4)(m-2) = 0$

$m_2 = 4 \quad m_1 = 2$

$C.F = Ae^{2x} + Be^{4x}$

$y = Ce^{4x}$

$\frac{dy}{dx} = 4Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Ce^{4x}$

$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

C is undefined

therefore $y = Cxe^{4x}$

$u = Cx \quad v = e^{4x}$

$\frac{du}{dx} = C \quad \frac{dv}{dx} = 4e^{4x}$

$\frac{dy}{dx} = Cx(4e^{4x}) + e^{4x}(C)$

$= 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 8Ce^{4x}$

$y = 16Cxe^{4x} + 8Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8Cxe^{4x} = 8e^{4x}$

$y = 16Cxe^{4x} + 8Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$2Ce^{4x} = 8e^{4x}$

$C = 4$

$y = 4xe^{4x}$

$C.S = Ae^{2x} + Be^{4x} + 4xe^{4x}$