Assignment 5

Oyedeji Siamah Oyedapo
18/ENG02/050
Elect/Elect Engr

JULY MONDAY 25

1) $\frac{dy}{dt} + 3y = e^{-2t}$ given at $t=0$, $y=2$, $y(0)=2$

$y^{(1)} + 3y^{(0)} = e^{-2t}$

$sy(s) - y(s) + 3y(s) = \frac{1}{s+2}$

$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$

$y(s)[s+3] = \frac{1}{s+2} + \frac{2}{1} = \frac{1+2(s+2)}{s+2} = \frac{1+2s+4}{s+2}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$A = \left.\frac{1+2s+4}{s+3}\right|_{s=-2} = \frac{1+2(-2)+4}{-2+3} = 1$

$B = \left.\frac{1+2s+4}{s+2}\right|_{s=-3} = \frac{1+2(-3)+4}{-3+2} = 1$

NOTES

$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = e^{-2t} + e^{-3t}$

d) $3\frac{dy}{dx} - 6y = \sin 2t$ at $t=0$, $y=1$, $y(0)$

$3y^{(1)} - 6y = \sin 2t$

$3(sy(s) - y(0)) - 6y(s) = \frac{2}{s+4}$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s+4}$

$y(s)[3s-6] = \frac{2}{s^2+4} + \frac{3}{1} = \frac{2+3(s^2+4)}{s^2+4}$

$y(s)[3s-6] = \frac{2+3s^2+12}{(s^2+4)} = \frac{3s^2+14}{(s^2+4)}$

$y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{(s^2+4)} + \frac{C}{(3s-6)}$

$C: \left.\frac{3s^2+14}{s^2+4}\right|_{s=2} = \frac{3(2)^2+14}{2^2+4} = \frac{13}{4}$

$3s^2+14 = (A+Bs)(3s-6) + C(s^2+4)$

$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C$

Comparing coefficients

NOTES $3 = 3B+C$

$3 = 3B + 13/4$ (where $C = 13/4$)

$3A - 6B = 0$

$3A = 6B$

$3A = 6 \times \frac{-1}{12}$ $A = -1/6$

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$$y(s) = \frac{-1/6 - 1/12\, s}{s^2+4} + \frac{13/4}{3s-6}$$

$$= \frac{-1/6}{s^2+4} - \frac{1/12\, s}{s^2+4} + \frac{13/4}{3s-6}$$

$$= \frac{-1/6}{s^2+4} - \frac{1/12\, s}{s^2+4} + \frac{13/4}{3s-6}$$

$$= -1/6 \cdot \frac{1}{s^2+2^2} - 1/12 \; s/s^2+2^2 + \frac{13}{4} \cdot \frac{1}{3(s-2)}$$

$$= -1/6 \cdot \frac{1}{s^2+2^2} - \frac{1}{12} \; s/s^2+2^2 + \frac{13}{4} \cdot \frac{1}{3(s-2)}$$

$$= \frac{-1}{6} \cdot \frac{1}{2}\left[\frac{2}{s^2+2^2}\right] - \frac{1}{12}\left[\frac{s}{s^2+2^2}\right] + \frac{13}{2}\left[\frac{1}{s-2}\right]$$

$$y(t) = -1/12 \sin 2t - 1/12 \cos 2t + \frac{13}{12} e^{2t}$$

$$y(t) = 1/2\left[-\sin 2t - \cos 2t + 13e^{2t}\right]$$

NOTES

$$y(t) = 1/12\left[\cancel{-\sin 2t}\; 13e^{2t} - \cos 2t - \sin 2t\right]$$

3) $\frac{dy}{dt} - 4y = 8$ given that $t = 0, y = 2, y(0$

$y^{(1)} - 4y = 8$

$sy(s) - y(0) - 4y(s) = 8/s$

$sy(s) - 2 - 4y(s) = 8/s$

$sy(s) - 2 - 4y(s) = 8/s$

$y(s)[s-4] = \frac{8}{s} + \frac{2}{1} = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$A: \frac{8+2s}{s-4}\Big|_{s=0} = \frac{8}{-4} = -2$

$B: \frac{8+2s}{s}\Big|_{s=4} = \frac{8+2(4)}{4} = 4$

$y(s) = \frac{-2}{s} + \frac{4}{s-4}$

$y(t) = -2 + 4e^{4t}$

NOTES

(A) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2t}$ at $t=0$, $y=2$, $y^{(1)}=1$

$y(0)=2$, $y'(0)=1$

$y^{(2)} - 2y^{(1)} + 5y = e^{2t}$

$$(s^2y(s) - sy(0) - y'(0)) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$$

$$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$$

$$y(s)[s^2 - 2s + 5] = \frac{1}{s-2} + \frac{2s}{1} - \frac{3}{1} = \frac{1 + 2s(s-2) - 3(s-2)}{s-2}$$

$$y(s)[s^2 - 2s + 5] = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2} = \frac{2s^2 - 7s + 7}{s-2}$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)}$$

NOTES

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5}$$

$$A = \frac{2s^2 - 7s + 7}{s^2 - 2s - 5}\Bigg|_{s=-2} = \frac{2(2)^2 - 7(2) + 7}{2^2 - 2(2) + 5} = \frac{1}{5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s - 2)$$

$$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

$$2 = A + B$$

$$2 = \frac{1}{5} + B = \quad B = 9/5$$

$$7 = 5A - 2C$$

$$7 = 5(1/5) - 2C$$

$$7 - 1 = -2C$$

$$C = -3$$

$$y(s) = \frac{1/5}{s-2} + \frac{9s/5 - 3}{s^2 - 2s + 5} - \frac{1}{5} \cdot \frac{1}{s-2} + \frac{\frac{9s}{5}}{(s+1)^2 + 4}$$

$$- \frac{3}{(s+1)^2}$$

$$y(s) = 1/5 \cdot 1/s-2 + \frac{9}{5} \frac{s-1+1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$$

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$$y(s) = 1/5 \cdot \frac{1}{s-2} + \frac{9}{5} \frac{s+1}{(s+1)^2 + 2^2} - \frac{1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$$

$$y(s) = \frac{1}{5} \cdot \frac{1}{s-2} + \frac{9}{5} \frac{s+1}{(s+1)^2 + 2^2} - \frac{4}{(s+1)^2 + 2^2} \cdot 2/2$$

NOTES

$$y(s) = \frac{1}{5} \cdot \frac{1}{(s-2)} + \frac{9}{5} \frac{s+1}{(s+1)^2 + 2^2} - \frac{2 \cdot 2}{(s+1)^2 + 2^2}$$

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$$y(t) = \frac{1}{5} e^{2t} + \frac{9}{5} e^{-t} \cos 2t - 2e^{-t} \sin 2t$$

$$y(t) = \frac{1}{5} \left[e^{2t} + 9e^{-t} \cos 2t - 10 e^{-t} \sin 2t \right]$$

$$= \frac{1}{5} \left[e^{2t} + e^{-t} (9 \cos 2t - 10 \sin 2t) \right]$$