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① $\frac{dy}{dt} + 3y = e^{-2t}$

$3y(s) - y(0) + 3y(s) = \frac{1}{s+2}$

when $t = 0$, $y = 2$

$sy(s) - 2 + 3y(s) = 1/s+2$

$y(s)(s+3) = 2(s+2)+1 / s+2$

$y(s) = A/s+2 + B/s+3$

$= \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

when $s = -2$

$A = -A + 5$

$A = 1$

when $s = -3$

$-6s+5=-B$

$B=1$

$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = e^{-2t} + e^{-3t}$

② $3\frac{dy}{dt} - 6y = \sin 2t$

$3[sy(s) - y(0)] - 6y(s) = \frac{2}{s^2+4}$

at $t=0$, $y=1$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3(1) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3 - 6y(s) = \frac{2}{s^2+4}$

$ys[3s-6] = \frac{3(s^2+4)+2}{}$

$$y(s)[3s-6) = \frac{3s^2+14}{s^2+4}$$

$$y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)}$$

$$y(s) = \frac{As+B}{s^2+4} + \frac{C}{3s-6}$$

$$y(s) = \frac{(As+B)(3s-6)+C(s^2+4)}{(s^2+4)(3s-6)}$$

$$3s^2+14 = (As+B)(3s-6)+C(s^2+4)$$

when $s=2$

$$26 = 8C$$

$$C = \frac{26}{8} = \frac{13}{4}$$

Comparing coefficients

$$3B = 0$$

$$B = 0$$

$3Ay = 0$

$Ay = 0$

$A = 0$

$\therefore \quad y(s) = \frac{13}{4}\left(\frac{1}{3s-6}\right)$

$= \frac{13}{4}\left[\frac{1}{3}\left(\frac{1}{s-2}\right)\right]$

$y(t) = \frac{13}{12} e^{2t}$

(3) $\frac{dy}{dt} - 4y = 8$

$sy(s) - y(0) - 4y(s) = 8/s$

At $t = 0, \quad y = 2$

$sy(s) - 2 - 4y(s) = 8/s$

$y(s)(s-4) = \frac{2s+8}{s}$

$$Y(s) = \frac{2s+8}{s(s-4)}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s-4}$$

$$= \frac{A(s-4) + Bs}{s(s-4)}$$

$$2s + 8 = A(s-4) + Bs$$

When $s = 0$

$$8 = -4A$$

$$A = -2$$

When $s = 4$

$$16 = 4B$$

$$B = 4$$

$$Y(s) = \frac{-2}{s} + \frac{4}{s-4}$$

$$y(t) = -2 + 4e^{4t}$$

(A) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$s^2y(s) - sy(0) - y'(0) - 2[sy(0) - y(0)] - 5y(s) = \frac{1}{s-2}$

At $t=0$, $y=2$, $y'=1$

$s^2y(s) - sy(0) - 1 - 2sy(s) + 4 - 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s - 2sy(s) + 3 - 5y(s) = \frac{1}{s-2}$

$y(s)[s^2 - 2s - 5] = \frac{1}{s-2} + 2s - 3$

$y(s) = \frac{1 + 2s(s-2) - 3(s-2)}{(s-2)(s^2-2s-5)}$

$y(s) = \frac{2s^2 - 7s - 7}{(s-2)(s+1.4)(s-3.4)}$

$y(s) = \frac{A}{(s-2)} + \frac{B}{(s+1.4)} + \frac{C}{(s-3.4)}$

$\Rightarrow \frac{A(s+1.4)(s-3.4) + B(s-2)(s-3.4) + C(s-2)(s+1.4)}{(s-2)(s+1.4)(s-3.4)}$

$2s^2 - 7s + 7 = A(s+1.4)(s-3.4) + B(s-2)(s-3.4) + C(s-2)(s+1.4)$