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$$\textcircled{1} \frac{dy}{dt} + 3y = e^{-2t}$$

at $t=0, y=2$

$$sY(s) - y_0 + 3(Y(s)) = \frac{1}{s+2}$$

$$sY(s) - 2 + 3Y(s) = \frac{1}{s+2}$$

$$Y(s)(s+3) = \frac{1}{s+2} + 2$$

$$Y(s)(s+3) = \frac{1 + 2(s+2)}{s+2}$$

$$Y(s)(s+3) = \frac{1 + 2s + 4}{s+2}$$

$$Y(s)(s+3) = \frac{2s+5}{s+2}$$

$$Y(s)(s+3) = \frac{2s+5}{s+2}$$

$$Y(s) = \frac{2s+5}{(s+2)(s+3)}$$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$2s+5 = A(s+3) + B(s+2)$$

when $s = -2$

$$2(-2)+5 = A(-2+3)$$

$$-4+5 = A$$

$$A = 1$$

when $s = -3$

$$2(-3)+5 = B(-3+2)$$

$$-6+5 = -B$$

$$-1 = -B$$

$$B = 1$$

$$Y(s) = \frac{1}{s+2} + \frac{1}{s+3}$$

$$y = e^{2t} + e^{-3t}$$

$$\textcircled{2} \quad 3 \frac{dy}{dt} - 6y = \sin 2t$$

at $t=0, y=1$

$$3(sY(s) - y_0) - 6Y(s) = \frac{2}{s^2+2^2}$$

~~$$3sY(s) - 3y_0 - 6Y(s) = \frac{2}{s^2+4}$$~~

$$Y(s)(3s-6) - 3 = \frac{2}{s^2+4}$$

$$Y(s) = \left(\frac{2}{s^2+4} + 3 \right) \div (3s-6)$$

$$Y(s) = \frac{2 + 3(s^2+4)}{(s^2+4)(3s-6)}$$

$$Y(s) = \frac{2 + 3s^2 + 12}{(s^2+4)(3s-6)}$$

$$Y(s) = \frac{3s^2 + 14}{(s^2+4)(3s-6)}$$

~~$$3s^2 + 14 = A(3s-6) + B(s^2+4)$$~~
 when $s =$

$$3s^2 + 14 = \frac{As+B}{s^2+4} + \frac{C}{3s-6}$$

$$3s^2 + 14 = As + B(3s-6) + C(s^2+4)$$

$$3s^2 + 14 = \underbrace{3As^2}_{\rightarrow} - \underbrace{6As}_{\rightarrow} + \underbrace{3Bs}_{\rightarrow} - \underbrace{6B}_{\rightarrow} + \underbrace{Cs^2 + 4C}_{\rightarrow}$$

3rd 14A

$$\begin{aligned}
 3A + C &= 3 \quad \dots \textcircled{i} \\
 -6A + 3B &= 0 \quad \dots \textcircled{ii} \\
 -6B + 4C &= 14 \quad \dots \textcircled{iii}
 \end{aligned}$$

$$C = 3 - 3A \quad \dots \textcircled{iv}$$

$$-6B + 4(3 - 3A) = 14$$

$$-6B + 12 - 12A = 14$$

$$-6B - 12A = 2 \quad \dots \textcircled{v}$$

$$-6A + 3B = 0$$

$$3B = +6A$$

$$B = +2A$$

$$-6(+2A) - 12A = 2$$

$$+12A + 12A = 2$$

$$+24A = 2$$

$$A = +\frac{1}{12}$$

$$B = +2 \times \frac{-1}{12}$$

$$B = -\frac{1}{6}$$

$$C = 3 - 3\left(\frac{-1}{12}\right)$$

$$C = 3 + \frac{3}{12}$$

$$\frac{3(12) + 3}{12}$$

$$C = \frac{36 + 3}{12}$$

$$C = \frac{39}{12} = \frac{13}{4}$$

$$Y(s) = \frac{\cancel{5} + 1}{12 \cdot 6} + \frac{13}{4} \left(\frac{1}{s^2 - 6} \right)$$

$$\frac{d^2 y}{dt^2} - 4y = 8 \quad t=0 \quad y = 2$$

$$s^2 x(s) - x(0) - 4x(s) = \frac{8}{s}$$

$$s^2 x(s) - 4x(s) = \frac{8}{s} + 2$$

$$x(s)(s-4) = \frac{8+2s}{s}$$

$$x(s) = \frac{10}{s-4} + \frac{2s}{s(s-4)}$$

$$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$8+2s = A(s-4) + Bs$$

$$8+2s = A(s-4) + Bs$$

$$s=4$$

$$16 = 4B$$

$$B = 4$$

$$s=0$$

$$8 = -4A$$

$$A = -2$$

$$= \frac{-2}{s} + \frac{4}{s-4}$$

$$= -2 + 4e^{4t}$$

$$3 \frac{dy}{dt} - 6y = \sin 2t \quad t=0 \quad y=1$$

$$3(sx(s) - x(0)) - 6x(s) = \frac{2}{s^2+4}$$

$$3sx(s) - 3x(0) - 6x(s) = \frac{2}{s^2+4}$$

$$3(sx(s) - 6x(s)) = \frac{2}{s^2+4} + 3$$

$$x(s)(3s-6) = \frac{2+3s^2+12}{s^2+4}$$

$$= \frac{3s^2+14}{(s^2+4)(3s-6)}$$

$$(s^2+4)(3s-6)$$

$$(iv) \quad \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t} \quad y=2 \quad y'=1$$

$$s^2y(s) - sy(0) - y'(s) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$$

$$s^2y(s) - 2s - 1 - 2sy(s) + 2 + 5y(s) = \frac{1}{s-2}$$

$$s^2y(s) - 2sy(s) + 5y(s) - 2s + 1 = \frac{1}{s-2}$$

$$s^2y(s) - 2sy(s) + 5y(s) = \frac{1}{s-2} + 2s + 1$$

$$\frac{1 + 2s(s-2) + 1(s-2)}{s-2}$$

$$8y(s)(s^2 - 2s + 5) = \frac{1 + 2s^2 - 4s + 1s - 2}{s-2}$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5}$$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + Bs + C(s-2)$$

$$s = 2$$

$$8 - 14 + 7 = 5A$$

$$A = 1/5$$

$$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

$$A + B = 2$$

$$B = 2 - 1/5$$

$$\frac{10 - 1}{5}$$

$$5$$

$$B = 9/5$$

$$-2A - 2B + C = -7$$

$$-2/5 - 18/5 + C = -7$$

$$-2 - 4 + C = -7$$

$$C = -3$$

$$\frac{1}{5} \frac{1}{(s-2)} + \frac{3s-3-1+1}{5(s^2-2s+5)}$$

$$= \frac{1}{5} e^{2t} + \frac{9s-1}{5(s^2-2s+5)} - \frac{3+1}{5(s^2-2s+5)}$$

v) ~~9s~~

$$\# \frac{9}{5} \times \frac{s-1}{s^2-s-1+4} - \frac{3+1}{s^2-s-1+4}$$

$$\# \frac{9}{5} \times \frac{s-1}{s(s-1)-1(s-1)+2^2} - \frac{3}{s(s-1)-1(s-1)+2^2}$$

$$\# \frac{9}{5} \left(\frac{s-1}{(s-1)^2+2^2} \right) - \frac{4}{(s-1)^2+2^2}$$

$$\# \frac{9}{5} \left(\frac{s-1}{(s-1)^2+2^2} \right) - 2 \left(\frac{2}{(s-1)^2+2^2} \right)$$

$$\frac{1}{5} e^{2t} \quad \# \frac{9}{5} e^t \cos 2t - 2 e^t \sin 2t$$

v)

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = e^{3t} \quad t=0 \quad y=0 \quad y'=2$$

$$s^2y(s) - sy(0) - y'(s) - 6(sy(s) - y(0)) + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 2 - 6sy(s) + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 6sy(s) + 8y(s) = \frac{1}{s-3} + 2$$

$$y(s)(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$s^2 - 4s - 2s + 8$
 $s(s-4) - 2(s-4)$

$$2s-5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-2)(s-3)$$

$$s = 3$$

$$1 = A(-1)$$

$$A = -1$$

$$s = 2$$

$$-1 = 2B$$

$$B = -1/2$$

$$s = 4$$

$$8-5 = 2C$$

$$3 = 2C$$

$$C = 3/2$$

$$\frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)}$$

$$-e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$