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Mechanical Engineering
15/ENG06/044
ENG 381

i) $\frac{dy}{dt} + 8y = e^{-2t}$

$y(t) + 8y(t) = e^{-2t}$

$\mathcal{L}[y'(t)] = sY(s) - Y(0)$

$\mathcal{L}[y(t)] = Y(s), \quad \mathcal{L}^{-1}(e^{-at}) = \frac{1}{s+a}$

$sY(s) - Y(s) + 8Y(s) = \frac{1}{s+2}$

$(s-8)Y(s) - 2 = \frac{1}{s+2}$

$(s-8)Y(s) = \frac{1}{s+2} + 2 = \frac{1+2s+4}{s+2} = \frac{2s+5}{s+2}$

$Y(s) = \frac{2s+5}{(s+2)(s-8)} = \frac{A}{s+2} + \frac{B}{s-8}$

$2(-2)+5 = A(-2-8) \Rightarrow A = \frac{1}{-5}$

$2(8)+5 = B(8+2) \Rightarrow B = \frac{11}{5}$

$Y(s) = \frac{-1}{5(s+2)} + \frac{11}{5(s-8)} = -\frac{1}{5}e^{-2t} + \frac{11}{5}e^{8t}$

ii) $3\frac{dy}{dt} - 6y = \sin 2t = 3y'(t) - 6y(t)$

$\mathcal{L}[y'(t)] = sY(s) - Y(0)$

$\mathcal{L}[y(t)] = Y(s)$

$\mathcal{L}[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$

$3sY(s) - 3Y(0) - 6Y(s) = \frac{2}{s^2+4}$

$Y(s)[3s-6] = \frac{2}{s^2+4} + 3 = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$

$Y(s) = \frac{3s^2+14}{(3s-6)(s^2+4)} = \frac{A}{3s-6} + \frac{B}{s^2+4}$

$$Y(s) = \frac{3s^2 + 14}{(3s-6)(s^2+4)} = \frac{A}{3s-6} + \frac{B}{s^2+4}$$

$$3s^2 + 14 = A(s^2+4) + B(3s-6)$$

$$4A - 6B = 14$$

$$-6B = 14 - 12$$

$$B = -1/3$$

$$Y(s) = \frac{B}{CBJ(s-2)} - \frac{1}{3(s^2+4)}$$

$$= e^{2t} - 1/6 \sin 2t$$

iii) $dy/dt - 4y = 8$

$$y'(t) - 4Y(s) = 8$$

$$y''(t) - 4y(t) = 8$$

$$sY(s) - y(0) - 4y(s) = 8/s$$

$$(s-4)Y(s) = 8/s + 2 = 8\frac{+2s}{s}$$

$$Y(s) = \frac{2s+8}{s(s-4)}$$

$$\frac{2s+8}{s(s-4)} = A/s + B/s-4$$

$$2s + 8 = A(s-4) + Bs$$

$$2(0) + 8 = A(0-4) \Rightarrow A = -2$$

$$2(4) + 8 = A(4-4) + B(4)$$

$$B = 4$$

$$L^{-1}\left[-2/s + 4/s-4\right] = -2 + 4e^{4t}$$

iv) $d^2y/dt^2 - 2dy/dt + 5y = e^{2t}$

$$y''(t) - 2y'(t) + 5y(t) = e^{2t}$$

$L[y''(t)] = s^2 Y(s) - sy(0) - y'(0)$

$L[y'(t)] = sY(s) - y(0)$

$L[y(t)] = Y(s)$

$s^2 Y(s) - sY(s) - y'(0) - 2sY(s) + 2Y(s) + 5Y(s) = \frac{1}{s-2}$

$[s^2 - 2s + 5)Y(s) + (2-5)y_0 - y'(0) = \frac{1}{s-2}$

$(s^2 - 2s + 5) Y(s) = \frac{1}{s-2} - (2-5) 2 / 1$

$= \frac{1}{s-2} + 2s - 4 + 1 = \frac{1}{s-2} + 2s - 3$

$= \frac{1 + (2s-3)(s-2)}{(s-2)} = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$

$Y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$

$2s^2 - 7s + 7 = \frac{A}{s-2} + \frac{B}{s^2 - 2s + 5}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$

$4 \cdot 2$

$-2A + B = -7$

$B = 2A - 7$

$B(2(2) - 7 = 4 - 7 = -3$

$= \frac{2}{s-2} \quad \frac{-3}{s-2s+5}$

$= 2e^{2t} - \frac{7}{3} \sin 2t$

v) $\frac{d^2y}{dx^2} - 6\frac{dy}{dt} + 8y = e^{3t}$

$s^2 Y(s) - 3y(0) - y'(0) - 6s Y(s) + 6y(0) + 8Y(s) = \frac{1}{s-3}$

$[s^2 - 6s + 8] Y(s) + 2 = \frac{1 + 2s - 6}{s-3} = \frac{2s-5}{s-3}$

$Y(s) = \frac{2s - 5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$

$2s - 5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-3)(s-2)$

$2s - 5 = A(s^2 - 6s + 5) + B(s^2 - 7s + 12) + C(s^2 - 5s + 11)$

$2(3) - 5 = A(3-2)(3-4) \Rightarrow A = -1$

$2(4) - 5 = C(4-3)(4-2) \Rightarrow C = 3/2$

$-6A - 7B - 3C = 7$

$-6(-1) - 7(B) - 3(3/2) = 2$

$-7B = 2 + 15/2 - 6 = 4 + 15 - 12 = 7/2 \Rightarrow B = -1/2$

$2^{-1}\left[\frac{1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)}\right]$

$= -e^{2t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$