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16/ENG01/003
CHEMICAL ENGINEERING
Assignment 5

1. $\frac{dy}{dt} + 3y = e^{-2t}$ at $t=0$, $y=2$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$

$sy(s) + 3y(s) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s) = \frac{1}{(s+2)(s+3)} + \frac{2}{s+3}$

$\frac{1}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$1 = A(s+3) + B(s+2)$

when $s = -3$

$1 = -B$

$B = -1$

when $s = -2$

$A = 1$

$\frac{1}{(s+2)(s+3)} = \frac{1}{s+2} - \frac{1}{(s+3)}$

$= L^{-1}\left\{\frac{1}{s+2}\right\} - L^{-1}\left\{\frac{1}{s+3}\right\} + L^{-1}\left\{\frac{2}{s+3}\right\}$

$= e^{-2t} - e^{-3t} + 2e^{-3t}$

2. $3\frac{dy}{dt} - 6y = \sin 2t$ given that at $t=0$, $y=1$

$3[sy(s) - y(0)] - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3(1) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3 - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 6y(s) = \frac{2}{s^2+4} + 3$

$y(s)[3s + 12 + 2] = \frac{3s^2+14}{s^2+4}$

$y(s)[3s+14] = 3s^2+4$

s^2+4

$\frac{3s^2+14}{s^2+4} = \frac{As+B}{} + \frac{C}{3s+6}$

$3s^2+14 = As+B(3s-6)+C(s^2+4)$

$3s^2+14 = 3As^2-6As+3Bs-6B+Cs^2+4C$

$3A+C=3$ --- ①

$-6A+3B=0$ --- ②

$-6B+4C=14$ --- ③

From ①, $C=3-3A$ --- ④

Substitute ④ into ③

$-6B+4(3-3A)=14$

$-6B+12-12A=14$

$-6B-12A=2$ --- ⑤

Also $-6A+3B=0$

$3B=6A$

$B=2A$ --- ⑥

Substitute eqn 6 into 5

$-6(2A)-12A=2$

$-12A-12A=2$

$A=-1/2$

$B=2A$

$=2(-1/2)$

$=-1/6$

$C=3-3A$

$=3-3(-1/12)$

$=\frac{3(12)+3}{12} = \frac{36+3}{12}$

$=\frac{39}{12}$

$=\frac{13}{4}$

$$Y(s) = \frac{-1}{12}\left[\frac{s+1}{(s^2+4)6}\right] + \frac{13}{4(3s-6)}$$

$$= \frac{s}{12} - \frac{1}{6}\left[\frac{1}{s^2+4}\right] + \frac{13}{4}\,\frac{1}{3s-6}$$

$$= \frac{1}{12}\cos 2t - \frac{1}{12}\sin 2t + \frac{13}{2}e^{2t}$$

3 $\frac{dy}{dt} - 4y = 8$ given that at $t=0$, $y=2$

$sy(s) - y(0) - 4y(s) = \frac{8}{s}$

$sy(s) - 2 - 4y(s) = 8/s$

$sy(s) - 4y(s) = \frac{8}{s} + 2$

$y(s) = \frac{2s+8}{s(s-4)}$

$(s-1)(s-1)$

$s^2 - s - s + 1$

$s^2 - 2s + 1$

$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$2s+8 = A[s-4] + Bs]$

when $s=0$

$8 = -4A$

$A = -2$

when $s=4$

$2(4)+8 = A[4-4] + 4B$

$8+8 = 4B$

$16 = 4B$

$B = 4$

$= \frac{-2}{s} + \frac{4}{s-4}$

$L^{-1}\left\{\frac{-2}{s}\right\} + L^{-1}\left\{\frac{4}{s-4}\right\}$

4 $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ at $t=0$, $y=2$, $y'=1$

$s^2y(s) - sy(0) - y'(0) - 2[sy(s) - y(0)] + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s - 1 - 2[sy(s)] + 4 + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2sy(s) + 5y(s) = 2s - 3 + \frac{1}{s-2}$

$y(s)[s^2 - 2s - 5] = 2s(s-2) - 3(s-2) + 1$

$s-2$

$y(s)[s^2 - 2s + 5] = \frac{2s^2 - 7s + 7}{s-2}$

$y(s) = \frac{2s^2 - 7s + 7}{(s^2 - 2s + 5)(s-2)} = \frac{A}{(s-2)} + \frac{B}{s^2 - 2s + 5}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$

when $s = 2$

$2(2)^2 - 7(2) + 7 = A(4 - 4 + 5) + B(2-2)$

$8 - 14 + 7 = 5A$

$1 = 5A$

$A = 1/5$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$A + B = 2$

$B = 2 - A$

$= 2 - 1/5$

$= 9/5$

$-2A - 2B + C = -7$

$-2(1/5) - 2(9/5) + C = -7$

$-2/5 - 18/5 + C = -7$

$-4 + C = -7$

$C = -3$

$$\frac{1}{5(s-2)} + \frac{9s-3-1+1}{5(s^2-2s+5)}$$

$$\frac{9s-1}{5(s^2-2s+5)} - \frac{3+1}{(s^2-2s+5)}$$

$$\frac{9}{5} \times \frac{s-1}{s^2-s-s+1+4} - \frac{3+1}{s^2-s-s+1+4}$$

$$\frac{9}{5} \times \frac{s-1}{s(s-1)-1(s-1)+2^2} - \frac{3}{s(s-1)-1(s-1)+2^2}$$

$$\frac{9}{5}\left(\frac{s-1}{(s-1)^2+2^2}\right) - \frac{4}{(s-1)^2+2^2}$$

$$\frac{9}{5}\left(\frac{s-1}{(s-1)^2+2^2}\right) - 2\left(\frac{2}{(s-1)^2+2^2}\right)$$

$$\frac{1}{5}e^{2t} - \frac{9}{5}e^{t}\cos 2t - 2e^{t}\sin 2t$$

5 $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ given that at $t=0$, $y=0$, $y'=2$

$s^2y(s) - sy(0) - y(0) - 6sy(s) + 6y(0) + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - 2 - 6sy(s) + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - 6sy(s) + 8y(s) = \frac{1}{s-3} + 2$

$y(s)(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{s-3}$

$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$

$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$

$2s-5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-2)(s-3)$

when $s=3$

$1 = A(-1)$

$1 = -A$

$A = -1$

when $s=2$

$-1 = 2B$

$B = -1/2$

when $s=4$

$8-5 = 2C$

$3 = 2C$

$C = 2/3$

$\frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{2}{3(s-4)}$

$L^{-1}\left\{\frac{1}{s-3}\right\} + L^{-1}\left\{\frac{-1}{2(s-2)}\right\} + L^{-1}\left\{\frac{2}{3(s-4)}\right\}$

$= -e^{3t} - 1/2e^{2t} + 2/3\,e^{4t}$