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MECH ENGR.

1. $dy/dt + 3y = e^{-2t}$ given that at $t=0, y=2$

$$L\{dy/dt\} + L\{3y\} = L\{e^{-2t}\}$$

$$sY(s) - Y(0) + 3Y(s) = \frac{1}{s+2}$$

at $t=0$ $y=2$

$$sY(s) - 2 + 3Y(s) = \frac{1}{s+2}$$

$$sY(s) + 3Y(s) = \frac{1}{s+2} + 2$$

$$Y(s)\{s+3\} = \frac{1+2s+4}{s+2}$$

$$Y(s) = \frac{2s+5}{(s+2)(s+3)}$$

Using partial fractions

$$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$2s+5 = A(s+3) + B(s+2)$$

at $s=-3$

$$-1 = -B \; ; \; B=1$$

at $s=-2$

$$1 = A$$

$$\therefore \; Y(s) = \frac{1}{s+2} + \frac{1}{s+3}$$

$$L^{-1}\{Y(s)\} = Y(t)$$

$$Y(t) = e^{-2t} + e^{-3t}$$

2. $3\,dy/dt - 6y = \sin 2t$ given that $t=0, y=1$

$$L\{3\,dy/dt - 6y\} = L\{\sin 2t\}$$

$$3(sY(s) - Y(0)) - 6Y(s) = \frac{2}{s^2+4}$$

at $t=0$, $y=1$

$$3sY(s) - 3 - 6Y(s) = \frac{2}{s^2+4}$$

$$Y_s(3s-6) = \frac{2}{s^2+4} + 3$$

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$$Y(s)(3s-6) = \frac{3s^2+14}{(s^2+4)}$$

$$Y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)}.$$

Using partial fractions.

$$Y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{(s^2+4)} + \frac{C}{3s-6}$$

$$3s^2+14 = A+Bs(3s-6) + C(s^2+4) \quad \text{---- } *$$

at $s = 2$

$$26 = 8C$$

$$C = 26/8 = 13/4$$

also, expanding $*$, we have.

$$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4.$$

Comparing coefficients.

$$3 = 3B + C$$

$$3 = 3B + 13/4 ; \quad B = -1/12$$

also $3A - 6B = 0$

$$3A = 6B.$$

$$A = 2B ; \quad A = -2/12 = -1/6$$

$$\therefore Y(s) = \frac{-1/6 - \{-1/12\}s}{s^2+4} + \frac{13/4}{3s-6}$$

$$= \frac{-1/6}{s^2+4} - \frac{1/12 s}{s^2+4} + \frac{13/4}{3s-6}.$$

$$= -\frac{1}{6} \cdot \frac{1}{s^2+4} - \frac{1}{12} \cdot \frac{s}{s^2+4} + \frac{13}{4} \cdot \frac{1}{3(s-2)}$$

$$L^{-1}\{Y(s)\} = Y(t)$$

$$= -\frac{1}{12}\sin 2t - \frac{1}{12}\cos 2t + \frac{13}{12}e^{2t}$$

$$Y(t) = \frac{1}{12}(e^{2t} - \sin 2t - \cos 2t).$$

3. $dy/dt - 4y = 8$ at $t=0,\ y=2$.

$y' - 4y = 8$

$sY(s) - Y(0) - 4Y(s) = \frac{8}{s}$

applying ~~can~~ IC

$sY(s) - 2 - 4Y(s) = \frac{8}{s}$

$sY(s) - 4Y(s) = \frac{8}{s} + 2$

$Y(s)(s-4) = \frac{8+2s}{s}$

$Y(s) = \frac{8+2s}{s(s-4)}$

Solving partial fractions

$Y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$8 + 2s = A(s-4) + B(s)$

at $s=4$

$16 = 4B,\quad B = 4$

at $s=0$

$8 = -4A,\quad A = -2.$

$Y(s) = \frac{-2}{s} + \frac{4}{s-4}$

$Y(t) = -2 + 4e^{4t}$

4 $d^2y/dx^2 - 2\,dy/dx + 5y = e^{2t}$ at $t=0,\ y=0,\ y'' = 1$

$\{s^2Y(s) - sY(0) - Y'(0)\} - 2\{sY(s) - Y(0)\} + 5Y(s) = 1/s-2$

$Y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)}$

Solving partial fractions

$\frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs+C}{s^2-2s+5}$

$[A] \Rightarrow \left.\frac{2s^2 - 7s + 7}{s^2 - 2s + 5}\right|_{s=2} = \frac{2(2)^2 - 7(2) + 7}{2^2 - 2(2) + 5} = \frac{1}{5}$

~~at~~ also; $2s^2 - 7s + 7 = A\{s^2 - 2s + 5\} + Bs+C\{s-2\}$

Comparing coefficients, $2 = A + B$

$2 = 1/5 + B,\quad B = 9/5,$

also $7 = 5A - 2C$

$7 = 5(1/5) - 2c \; ; \quad c = -3.$

$$Y_{(s)} = \frac{1/5}{(s-2)} + \frac{9/5 s - 3}{s^2 - 2s + 5}$$

$$= \frac{1}{5}\left(\frac{1}{s-2}\right) + \frac{\frac{9}{5} s}{(s+1)^2 + 4} - \frac{3}{(s+1)^2}$$

$$L^{-1}\{Y_s\} = Y_{(t)}$$

$$= \frac{1}{5} e^{2t} + \frac{9}{5} e^{-t} \cos 2t - 2e^{-t} \sin 2t$$

$$= \frac{1}{5}\left(e^{2t} + e^{-t}\{9\cos 2t - 10 \sin 2t\}\right)$$

5. $d^2y/dt^2 - 6\, dy/dt + 8y = e^{3t}$ at $t = 0, y = 0, y' = 2$

$L\{\cancel{Y_{(s)}}\}$

$$s^2 Y_{(s)} - s Y_{(0)} - Y'_{(0)} - 6\{sY_{(s)} - Y_{(0)}\} + 8Y_{(s)} = \frac{1}{s-3}$$

applying IC

$$Y_{(s)}\{s^2 - 6s + 8\} = \frac{1 + 2s - 6}{s - 3}$$

$$Y_{(s)} = \frac{2s - 5}{(s-3)(s-2)(s-4)}$$

Using Partial fractions.

$$Y_{(s)} = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{(s-3)} + \frac{B}{(s-2)} + \frac{C}{(s-4)}$$

$$2s - 5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-3)(s-2)$$

at $s = 2$

$-1 = 2B \; ; \quad B = -1/2$

also at $s = 4$

$3 = 2C \; , \quad C = 3/2$

also at $s = 3$.

$1 = -A \; , \quad A = -1$

$$Y_{(s)} = \frac{-1}{(s-3)} - \frac{1}{2}\frac{1}{(s-2)} + \frac{3}{2(s-4)}$$

$$L^{-1}\{Y_{(s)}\} = Y_{(t)}$$

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