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i) $\frac{dy}{dx} + 3y = e^{-2t}$ $t=0$ & $y=0$

$y' + 3y = e^{-2t}$

$y' = sy_{(s)} - y_{(0)}$

$sy_{(s)} - y_{(0)} + 3y_{(s)} = \frac{1}{s+2}$

$sy_{(s)} - 2 + 3y_{(s)} = \frac{1}{s+2}$

$sy_{(s)} + 3y_{(s)} = \frac{1}{s+2} + 2$

$sy_{(s)} + 3y_{(s)} = \frac{2s+5}{s+2}$

$y_{(s)}(s+3) = \frac{2s+5}{s+2}$

$y_{(s)} = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{(s+3)}$

$\frac{A}{s+2}\Big|_{s=-2}$ $\frac{2(-2)+5}{-2+3} = \frac{-4+5}{1} = -1$

$\frac{B}{s+3}\Big|_{s=-3}$ $\frac{2(-3)+5}{-3+2} = \frac{-6+5}{-1} = 1$

$y(t) = L^{-1}\left[\frac{A}{s+2} + \frac{B}{s+3}\right]$

$= L^{-1}\left[\frac{-1}{s+2} + \frac{1}{s+3}\right]$

$y(t) = -e^{-2} + e^{-3}$

$= e^{-3} - e^{-2}$

ii) $3\frac{dy}{dt} - 6y = \sin 2t$ at $t=0$ $y=1$

$3y' - 6y = \sin 2t$

$3(sy_{(s)} - y_{(0)}) - 6y_{(s)} = \frac{1}{s^2+4}$

$3(sy_{(s)} - 1) - 6y_{(s)} = \frac{2}{s^2+4}$

$3sy_{(s)} - 3 - 6y_{(s)} = \frac{2}{s^2+2^2}$

$3sy_{(s)} - 6y_{(s)} = \frac{2}{s^2+4} + 3$

$3sy_{(s)} - 6y_{(s)} = \frac{3s^2+14}{s^2+4}$

$y_{(s)} = \frac{3s^2+14}{(3s-6)(s+2)}$

$= \frac{A}{(3s-6)} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$

$\frac{A(s+2)^2 + B(s+2)(3s-6) + C(3s-6)}{(3s-6)(s+2)^2}$

$A(s+2)^2 + B(s+2)(3s-6) + C(3s-6$

If $s=-2$

$c12 = 0$

iii) $\frac{dy}{dt} - 4y = 8$ $t=0$ $y=2$

$y' - 4y = 8$

$sy_{(s)} - y_{(0)} - 4y_{(s)} = \frac{8}{s}$

$sy_{(s)} - 2 - 4y_{(s)} = \frac{8}{s}$

$sy_{(s)} - 4y_{(s)} = \frac{8}{s} + 2$

$sy_{(s)} - 4y_{(s)} = \frac{8+2}{s}$

$y_{(s)}(s-4) = \frac{2s+8}{s}$

$y_{(s)} = \frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$\frac{A}{s}\Big|_{s=0}$ $\frac{2(0)+8}{0-4} = \frac{8}{-4} = -2$

$\frac{B}{s-4}\Big|_{s=4}$ $\frac{2(4)+8}{8} = \frac{16}{4} = 4$

$y(t) = L^{-1}(y_{(s)})$

$= L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right]$

$= L^{-1}\left[\frac{-2}{s}\right] + L^{-1}\left[\frac{4}{s-4}\right]$

$= -2 + 4e^{4t}$

$= 4e^{4t} - 2$

iv) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$t=0$ $y=1$ $y'=1$

$y'' - 2y' + 5y = e^{2t}$

$sy_{(s)} = sp_{(0)} - y'_{(0)} - 2(sy_{(s)} - y_0) + 5y_0 = \frac{1}{s-2}$

$$s^2 y_{(s)} - 2s - 1 - 2s y_{(s)} - 2 + 5y_{(s)} = \frac{1}{s-2}$$

$$s^2 y_{(s)} - 2s y_{(s)} + 5y_{(s)} - 2s = \frac{1}{s-2} + 3$$

$$s^2 y_{(s)} - 2s y_{(s)} + 5y_{(s)} - 2s = \frac{3s-5}{s-2}$$

$$y_{(s)} (s^2 - 4s + 5) = \frac{3s-5}{s-2}$$

$$y_{(s)} = \frac{3s-5}{(s^2+4s+5)(s-2)} = \frac{A}{s-3} + \frac{Bs+C}{s^2+4s+5}$$

$$s^2 - 4s + 5$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{4 \pm \sqrt{16-20}}{2}$$

$$\frac{4 + \sqrt{-4}}{2} \qquad \frac{4 \pm 2}{2}$$

$$2 \pm 3$$

5) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ at $t=0, y=2, y'=1$

solution

$L\{y''\} = s^2Y_{(s)} - sY_{(0)} - y'_{(0)}$

$L\{-2y'\} = -2[sY_{(s)} - Y_{(0)}]$

$L\{5y\} = 5Y_{(s)}$

$L\{e^{2t}\} = \frac{1}{s-2}$

$s^2Y_{(s)} - sY_{(0)} - y'_{(0)} - 2[sY_{(s)} - Y_{(0)}] + 5Y_{(s)} = \frac{1}{s-2}$

$s^2Y_{(s)} - 2sY_{(s)} + 5Y_{(s)} - s(2) - 1 + 2(2) = \frac{1}{s-2}$

$s^2Y_{(s)} - 2sY_{(s)} + 5Y_{(s)} - 2s - 1 + 4 = \frac{1}{s-2}$

$Y_{(s)}[s^2 - 2s + 5] = \frac{1}{s-2} + 2s - 3$

$Y_{(s)}[s^2 - 2s + 5] = \frac{(2s-3)(s-2)}{s-2}$

$Y_{(s)} = \frac{(2s-3)(s-2)}{(s-2)} \div [s^2 - 2s + 5] = \frac{1 + 2s^2 - s + 6}{(s-2)(s^2 - 2s + 5)}$

$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{(s-2)} + \frac{Bs + C}{(s^2 - 2s + 5)}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$, (comparing coefficients)

$A + B = 2$ — (1)

$-2A - 2B + C = -7$ — (2)

$5A - 2C = 7$ — (3)

from (1)

$B = 2 - A$

$-2A - 2(2-A) + C = -7$

$-2A + 4 + 2A + C = -7$

$C = 3$

from (3)

$5A - 2(-3) = 7$

$5A = 7-6$

$5A = 1$

$A = 1/5$

$\therefore A + B = 2$

$1/5 + B = 2$

$B = 2 - \frac{1}{5} = \frac{9}{5}$

$$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{1/5}{(s-2)} + \frac{9/5 s - 3}{(s^2-2s+5)}$$

$$= \frac{1/5}{s-2} + \frac{9/5 s}{(s^2-2s+5)} - \frac{3}{(s^2-2s+5)}$$

$$= \frac{1}{5(s-2)} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2+2^2} - \frac{1 \times 2/2}{(s-1)^2+2^2}\right] - \frac{3}{2}\left[\frac{2}{(s-1)+2^2}\right]$$

$$= L^{-1}\left\{\frac{1/5}{(s-2)} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2+2^2} + \frac{1}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right] - \frac{3}{2}\left(\frac{2}{(s-1)^2+2}\right)\right.$$

$$= \frac{1}{5}e^{2t} + \frac{9}{5}\left[e^t \cos 2t + \frac{1}{2} e^t \sin 2t\right] - \frac{3}{2}\left(e^t \sin 2t\right)$$

4) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0$, $y=0$, $y'=2$

Solution

$y'' - 6y' + 8y = e^{3t}$

$L\{y''\} = s^2 Y(s) - sY(0) - Y'(0)$

$L\{-6y'\} = -6[sY(s) - Y(0)]$

$L\{8y\} = 8Y(s)$

$L\{e^{3t}\} = \frac{1}{s-3}$

$\therefore s^2 Y(s) - sY(0) - Y'(0) - 6[sY(s) - Y(0)] + 8Y(s) = \frac{1}{s-3}$

$s^2 Y(s) - s(0) - 2 - 6sY(s) - 0 + 8Y(s) = \frac{1}{s-3}$

$s^2 Y(s) - 6sY(s) + 8Y(s) = \frac{1}{s-3} + 2$

$Y(s)[s^2 - 6s + 8] = \frac{1}{s-3} + 2 = \frac{1+2(s-3)}{s-3} = \frac{1+2s-6}{s-3} = \frac{2s-5}{s-3}$

$Y(s) = \frac{2s-5}{s-3} \div [s^2-6s+8] = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{s-3} + \frac{Bs+C}{(s^2-6s+8)}$

$2s-5 = A(s^2-6s+8) + (Bs+C)(s-3)$

$2s-5 = As^2 - 6As + 8A + Bs^2 - 3Bs + Cs - 3C$

Comparing coefficients

$A + B = 0$ —— (1)

$-6A - 3B + C = 2$ —— (2)

$8A - 3C = -5$ —— (3)

$B = -A$ from (1)

$-6A - 3(-A) + C = 2$

$-6A + 3A + C = 2$

$-3A + C = 2$ —— (4) $\times -3$

~~$C = 2+3A$~~

$8A - 3C = -5$.

$8A - 3(2+3A) = 2$ $\quad 9A - 3C = -6$ —— (5)

$8A - 6 - 9A = 2$ $\quad$ equ (5) - (4)

$A = -1$

$\therefore A = -B$ or $B = -A$

$B = -(-1) = 1$

from (3)

$8(-1) - 3c = -5$

$-8 + 5 = 3c$

$-3 = 3c$

$c = -1$

$$\therefore \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{1}{s-3} + \frac{s-1}{(s^2-6s+8)}$$

$$\frac{s-1}{s^2-6s+8} = \frac{A}{s-2} + \frac{B}{s-4}$$

$s - 1 = A(s-4) + B(s-2)$

If $s = 2$

$2 - 1 = A(-2)$

$1 = -2A$

$A = -1/2$

If $s = 4$

$4 - 1 = B(4-2)$

$3 = 2B$

$B = 3/2$

$$\frac{s-1}{(s-2)(s-4)} = \frac{-1/2}{s-2} + \frac{3/2}{s+4}$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{-1}{s-3} - \frac{1/2}{s-2} + \frac{3/2}{s-4}\right\}$$

$$y = -e^{3t} - \tfrac{1}{2}e^{2t} + \tfrac{3}{2}e^{4t}$$

$$y = \frac{1}{2}\left[2e^{3t} + e^{2t} - 3e^{4t}\right]$$

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