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15/ENG01/008
CHEMICAL ENGINEERING
ENG381

1. $\frac{dy}{dt} + 3y = e^{-2t}$, given that at $t = 0$, $y = 2$

$L\{dy/dt\} = sy(s) - y(0)$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) - 2 + 3y(s) = 1/s+2$

$sy(s) + 3y(s) - 2 = 1/s+2$

$y(s)[s+3] = \frac{1}{s+2} + 2$

$y(s)[s+3] = \frac{1 + 2(s+2)}{(s+2)}$

$y(s) = \frac{1 + 2s + 4}{(s+2)(s+3)} = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$2s + 5 = A(s+3) + B(s+2)$

when $s + 3 = 0$

$s = -3$

$2(-3) + 5 = A(-3+3) + B(-3+2)$

$-6 + 5 = A(0) + B(-1)$

$-1 = -B$

$B = 1$

when $s + 2 = 0$

$s = -2$

$2(-2) + 5 = A(-2+3) + B(-2+2)$

$-4 + 5 = A(1) + B(0)$

$1 = A$

$\therefore A = 1$

$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$

$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{1}{s+2}\right\} + L^{-1}\left\{\frac{1}{s+3}\right\}$

$= e^{-2t} + e^{-3t}$

2. $3\frac{dy}{dt} - 6y = \sin 2t$, given that at $t=0$, $y=1$

$3L\{\cancel{3}\,dy/dt\} = 3\{sy(s) - y(0)\}$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3 - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 6y(s) - 3 = \frac{2}{s^2+4}$

$y(s)(3s-6) = \frac{2}{s^2+4} + 3$

$y(s)(3s-6) = \frac{2+3(s^2+4)}{s^2+4} = \frac{2+3s^2+4}{(s+2)^2}$

$y(s) = \frac{2+3s^2+12}{(s+2)^2(3s-6)} = \frac{3s^2+14}{(s+2)^2(3s-6)}$

$\frac{3s^2+14}{(s+2)^2(3s-6)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{3s-6}$

$3s^2+14 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$

~~when $s+2=0$ $s=-2$~~

~~$3(-2)^2+14 = A(0) + B[3(-2)-6] + C(0)$~~

~~$12+14 = B(-12)$~~

~~$26 = -12B$~~

~~$B = -$~~

$2+3(s^2+2)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$

$2+3s^2+12s+12 = As^2 - A2 + 3Bs - 6B + Cs^2 + 4Cs + 4C$

$3A + C = 3$

$3B + 4C = 12$

$-12A - 6B + 4C = 14$

$3A = 3 - C$

$A = \frac{3-C}{3}$

$3B + 4C = 12$

$-12\left(\frac{3-C}{3}\right) - 6B + 4C = 14$

$$-12+4C-6B+4C=14$$

$$-6B+8C=26$$

$$-18B-24C=-72$$

$$-18B-24C=78$$

$$-48C=-150$$

$$C=25/8$$

$$3B=12-4(25/8)$$

$$B=-1/6$$

$$3A=3-C$$

$$A=\frac{3-(25/8)}{3}$$

$$A=-1/24$$

$$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)}=\frac{-1/24}{s-2}-\frac{1/6}{(s+2)^2}+\frac{25/8}{3s-6}$$

$$L^{-1}\{y(s)\}=L^{-1}\left\{\frac{-1/24}{(s+2)}-\frac{1/6}{(s+2)^2}+\frac{25/8}{(3s-6)}\right\}$$

$$y=-1/24\,e^{-2t}-1/6\,te^{-4t}+\frac{25}{24}e^{3t}$$

$$y=-1/6\left\{\frac{1}{4}e^{-2t}-te^{-4t}+25/4\,e^{3t}\right\}$$

3. $\frac{dy}{dt} - 4y = 8$, given that at $t=0$, $y=2$

$sy(s) - y(0) - 4y(s) = \frac{8}{s}$

$sy(s) - 4y(s) - 2 = 8/s$

$y(s)(s-4) = \frac{8}{s} + 2$

$\frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)}$

$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

when $s-4=0$ $\therefore s=4$

$8+2s = A(s-4) + Bs$

$8+2(4) = A(4-4) + B(4)$

$16 = A(0) + 4B$

$B = 4$

$s = 0$

$8+2(0) = A(0-4) + B(0)$

$8 = A(-4)$

$A = -2$

$\frac{8+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{s-4}$

$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{-2}{s}\right\} + 4L^{-1}\left\{\frac{1}{s-4}\right\}$

$= -2 + 4e^{4t}$

4. $\frac{d^2y}{dt^2} - \frac{2dy}{dt} + 5y = e^{2t}$, given that at $t=0$, $y=2$, $y'=1$

$s^2y(s) - sy(0) - y'(0) - 2sy(s) + 2y(0) + 5y(s) = 1/s-2$

$s^2y(s) - 2sy(s) + 5y(s) - 2s - 1 + 4 = 1/s-2$

$y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 3$

$\frac{1 + 2s(s-2) - 3(s-2)}{s-2}$

$= \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$

$y(s)(s^2 - 2s+5) = \frac{2s^2 - 7s + 7}{s-2}$

$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)}$

$\frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + Bs + C(s-2)$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

s^2 : $A + B = 2$ -- (1)

s : $-2A - 2B$ ~~...~~ $+ C = -7$ -- (2)

: $5A - 2C = 7$ -- (3)

From (1)

$B = 2 - A$

From (2)

$-2A - 2(2-A) + C = -7$

$-2A - 4 + 2A + C = -7$

$-4 + C = -7$

$C = -7 + 4$ $\therefore C = -3$

From 3

$5A - 2(-3) = 7$

$5A + 6 = 7$

$5A = 7 - 6$

$A = 1/5$

Recall

$B = 2 - A$

$B = 2 - 1/5$

$B = 9/5$

$$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{1/5}{(s-2)} + \frac{9/5\, s - 3}{(s^2-2s+5)}$$

$$= \frac{\frac{1}{5}}{s-2} + \frac{9/5\, s}{s^2-2s+5} - \frac{3}{s^2-2s+5}$$

$$= \frac{1/5}{s-2} + \frac{9}{5}\left[\frac{s-1+1}{(s-1)^2+4}\right] - \frac{3}{2}\left[\frac{2}{(s-1)^2+4}\right]$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{1/5}{s-2}\right\} + \frac{9}{5}L^{-1}\left\{\frac{s-1}{(s-1)^2+2^2} - \frac{1\times 2/2}{(s-1)^2+2^2}\right\}$$

$$-\frac{3}{2}L^{-1}\left\{\frac{2}{(s-1)^2+2^2}\right\}$$

$$L^{-1}\left\{\frac{1/5}{s-2} + \frac{9}{5}\left\{\frac{s-1}{(s-1)^2+2^2} + \frac{1}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right]\right. - \frac{3}{2}\left\{\frac{2}{(s-1)\ldots}\right.$$

$$y = \frac{1}{5}e^{2t} + \frac{9}{5}\left[e^{t}\cos 2t + \frac{1}{2}e^{t}\sin 2t\right] - \frac{3}{2}\left[e^{t}\sin\ldots\right.$$

5. $\frac{d^2y}{dt^2} - \frac{6dy}{dt} + 8y = e^{3t}$ given at $t=0, y=0, y'=2$

$$s^2y(s) - sy(0) - y'(0) - 6sy(s) + 6y(0) + 8y(s) = \ldots$$

$$s^2y(s) - 6sy(s) + 8y(s) - 2 = 1/s-3$$

$$y(s)(s^2-6s+8) = 1/s-3 + 2$$

$$y(s)(s^2-6s+8) = \frac{1+2(s-3)}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{s-3} + \frac{Bs+C}{s^2-6s+8}$$

$$2s-5 = A(s^2-6s+8) + Bs+C(s-3)$$

$$2s-5 = As^2 - 6As + 8A + Bs^2 - 3Bs + Cs - 3C$$

s^2: $A + B = 0$ --- (1)

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$S: -6A - 3B + C = 2 \quad - - (2)$

$\therefore 8 \cdot A - 3C = -3 \quad - - (3)$

$B = -A$ from (1)

$-6A + 3A + C = 2$

$-3A + C = 2$

$8A - 3C = 5$

$9A - 3C = -6$

$8A - 3C = -5$

$A = -1$

$B = 1$

From

$C = 2 - 3 \quad \therefore C = -1$

$$\frac{2S-5}{(S-3)(S^2-6S+8)} = \frac{-1}{S-3} + \frac{S-1}{S^2-6S+8}$$

$$\frac{2S-5}{(S-3)(S^2-6S+8)} = \frac{-1}{S-3} + \frac{S-1}{(S-2)(S-4)}$$

$$\frac{S-1}{(S-2)(S-4)} = \frac{A}{S-2} + \frac{B}{S-4}$$

$S - 1 = A(S-4) + B(S-2)$

$S - 1 = AS - 4A + BS - 2B$

$A + B = 1 \quad \times -4$

$-4A - 2B = -1 \quad \times 1$

$-4A - 4B = -4$

$-4A - 2B = -1$

$-2B = -3$

$B = \frac{3}{2}$

$A = -\frac{1}{2}$

$$\frac{S-1}{(S-2)(S-4)} = \frac{-1/2}{S-2} + \frac{3/2}{S-4}$$

$$\frac{2s-5}{(s-2)(s-4)} = \frac{-1}{s-3} + \left(\frac{-1/2}{s-2} + \frac{3/2}{s-4}\right)$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{-1}{s-3} - \frac{1/2}{s-2} + \frac{3/2}{s-4}\right\}$$

$$y = -e^{3t} - \tfrac{1}{2}e^{2t} + \tfrac{3}{2}e^{4t}$$

$$y = -\tfrac{1}{2}\left[2e^{3t} + e^{2t} - 3e^{4t}\right\}$$

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