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15/ENGG01/005

$$\frac{dy}{dt} + 3y = e^{-2t}$$

$$sY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$$

$$Y(s)(s+3) = \frac{1}{s+2} + 2$$
$$\frac{1 + 2(s+2)}{s+2}$$

$$Y(s) = \frac{2s+5}{(s+2)(s+3)}$$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$2s+5 = A(s+3) + B(s+2)$$
$$s = -3$$

$$-6+5 = -B$$

$$B = 1$$

$$s = -2$$

$$4-3 = A$$

$$1 = A$$

$$A = 1$$

$$z = e^{-3t} + e^{-2t}$$

A = -2

$$= -\frac{2}{s} + \frac{4}{s-4}$$

$$= -2 + 4e^{4t}$$

$$3 \frac{dy}{dt} - 6y = \sin 2t \quad t=0 \quad y=1$$

$$3(sx(s) - x(0)) - 6x(s) = \frac{2}{s^2+4}$$

$$3sx(s) - 3x(0) - 6x(s) = \frac{2}{s^2+4}$$

$$3(5x(s) - 6x(s)) = \frac{2}{s^2+4} + 3$$

$$x(s)(3s-6) = \frac{2+3s^2+12}{s^2+4}$$
$$= \frac{3s^2+14}{(s^2+4)(3s-6)}$$

$$s^2 y(s) - s y(0) - y'(0) = \frac{3s^2 + 14}{(s^2 + 4)(3s - 6)}$$

$$3s - 6 = 0 \\ s = \frac{6}{3}$$

$$\frac{3s^2 + 14}{(s^2 + 4)(3s - 6)} = \frac{A}{s^2 + 4} + \frac{B}{3s - 6} + \frac{C}{3s - 6}$$

$$3s^2 + 14 = \frac{A(s^2 + 4) + B(s^2 + 4) + C(s^2 + 4)(3s - 6)}{(s^2 + 4)(3s - 6)}$$

$$s = 2$$

$$12 + 14 = 8C$$

$$26 = 8C$$

$$C = \frac{13}{4}$$

$$3As^2 - 6As + 3Bs - 6B + (s^2 + 4)C$$

$$3 = 3A + C$$

$$\frac{3 - 13}{4} = 3A$$

$$\frac{12 - 13}{4} = 3A$$

$$-\frac{1}{4} = 3A \quad A = -\frac{1}{12}$$

$$-6A + 3B = 0$$

$$-6(-\frac{1}{12}) + 3B = 0$$

$$\frac{1}{2} = -3B$$

$$-\frac{1}{6} = B$$

$$= \frac{1}{12} \left[\frac{s + 1}{(s^2 + 4)6} \right] + \frac{13}{4(3s - 6)}$$

$$= \frac{1}{12} s - \frac{1}{6} \left[\frac{1}{s^2 + 4} \right] + \frac{13}{4(3s - 6)}$$

$$= \frac{1}{12} \cos 2t - \frac{1}{12} \sin 2t + \frac{13}{12} e^{2t}$$

(iv)

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t} \quad y=2 \quad y'=1$$

$$s^2y(s) - sy(0) - y'(s) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$$

$$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$$

$$s^2y(s) - 2sy(s) + 5y(s) - 2s + 3 = \frac{1}{s-2}$$

$$s^2y(s) - 2sy(s) + 5y(s) = \frac{1}{s-2} + 2s + 3$$

$$\frac{1 + 2s(s-2) + 3(s-2)}{s-2}$$

$$y(s)(s^2 - 2s + 5) = \frac{1 + 2s^2 - 4s + 3s - 6}{s-2}$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + Bs + C(s-2)$$

$$8 - 14 + 7 = 5A$$

$$A = 1/5$$

$$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

$$A + B = 2$$

$$B = 2 - 1/5$$

$$\frac{10-1}{5}$$

$$B = 9/5$$

$$-2A - 2B + C = -7$$

$$-2/5 - 18/5 + C = -7$$

$$-4 + C = -7$$

$$C = -3$$

$$\frac{1}{5} + \frac{9s-3-1+1}{5(s^2-2s+5)}$$

$$= \frac{1}{5} e^{2t} + \frac{9s-1}{5(s^2-2s+5)} - \frac{3+1}{5(s^2-2s+5)}$$

v) $\frac{9s}{5}$

$$+ \frac{9}{5} \times \frac{s-1}{s^2-s-s+1+4} - \frac{3+1}{s^2-s-s-1+4}$$

$$+ \frac{9}{5} \times \frac{s-1}{s(s-1)-1(s-1)+2^2} - \frac{3}{s(s-1)-1(s-1)+2^2}$$

$$+ \frac{9}{5} \left(\frac{s-1}{(s-1)^2+2^2} \right) - \frac{4}{(s-1)^2+2^2}$$

$$+ \frac{9}{5} \left(\frac{s-1}{(s-1)^2+2^2} \right) - 2 \left(\frac{2}{(s-1)^2+2^2} \right)$$

$$\frac{1}{5} e^{2t} + \frac{9}{5} e^t \cos 2t - 2 e^t \sin 2t$$

v)

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = e^{3t} \quad t=0 \quad y=0 \quad y'=2$$

$$s^2y(s) - sy(0) - y'(s) - 6(sy(s) - y(0)) + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 2 - 6sy(s) + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 6sy(s) + 8y(s) = \frac{1}{s-3} + 2$$

$$y(s)(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{s-3} + \frac{B}{s^2-6s+8} + \frac{C}{s-4}$$

$$s-5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-2)(s-3)$$

$$= A(s-1)$$

$$A = -A \quad s=2$$

$$-A = 2B$$

$$B = -1/2 \quad s=4$$

$$s-5 = 2C$$

$$3 = 2C$$

$$C = 2/3$$

$$= \frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{2}{3(s-4)}$$
$$= e^{3t} - 1/2 e^{2t} + 2/3 e^{4t}$$