HAASTRUP OLUWADOYINSOLA MERCY

15/ENG06/032

MECHANICAL ENGINEERING

ASSIGNMENT

ENG 381

i) $\frac{dy}{dt} + 3y = e^{-2t}$, given that at $t = 0$, $y = 2$.

$$SY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$$

$$SY(s) - 2 + 3Y(s) = \frac{1}{s+2}$$

$$SY(s) + 3Y(s) = \frac{1}{s+2} + 2$$

$$Y(s)(s+3) = \frac{1 + 2s + 4}{s+2} = \frac{2s+5}{s+2}$$

$$Y(s) = \frac{2s+5}{(s+2)(s+3)}$$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$2s + 5 = A(s+3) + B(s+2)$$

let $s = -3$

$$-1 = -B \; ; \; B = 1$$

let $s = -2$

$$1 = A$$

$$Y(s) = L^{-1}\left[\frac{1}{s+2} + \frac{1}{s+3}\right]$$

$$Y(s) = e^{-2t} + e^{-3t}$$

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ii) $3\frac{dy}{dt} - 6y = \sin 2t$ given that at $t=0, y=1$

$3(sY(s) - y(0)) - 6Y(s) = \frac{2}{s^2+4}$

$3sY(s) - 3y(0) - 6Y(s) = \frac{2}{s^2+4}$

$3sY(s) - 3 - 6Y(s) = \frac{2}{s^2+4}$

$3sY(s) - 6Y(s) = \frac{2}{s^2+4} + 3$

$Y(s)(3s-6) = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$

$Y(s) = \frac{3s^2+14}{(3s-6)(s^2+4)} \equiv \frac{A}{(3s-6)} + \frac{Bs+C}{s^2+4}$

$3s^2+14 = A(s^2+4) + (Bs+C)(3s-6)$

$3s^2+14 = As^2+4A+3Bs^2-6Bs+3Cs-6C$

$3s^2+14 = (A+3B)s^2+(-6B+3C)s+(4A-6C)$

Comparing coefficients.

$A+3B = 3$ ... (i)

$-6B+3C = 0$ ... (ii)

$4A-6C = 14$ ... (iii)

$6B = 3C$; $B = \frac{1}{2}C$ subst in (i)

$A + \frac{3}{2}C = 3$; $A = 3 - \frac{3}{2}C$: subst in (iii)

$4(3-\frac{3}{2}C) - 6C = 14$

$12 - 6C - 6C = 14$

$-12C = 2$; $C = -\frac{1}{6}$.

$B = \frac{1}{2} \times -\frac{1}{6} = -\frac{1}{12}$. ; subst B in i.

$A - \frac{1}{4} = 3$

$A = 3\frac{1}{4}$; $= 13/4$ subst. A, B, C

$Y(s) = L^{-1}\left[\frac{13/4 + -\frac{1}{12}s + (-\frac{1}{6})}{(3s-6) \quad s^2+4}\right]$

$Y(s) = L^{-1}\left[\frac{13}{4\cdot 3(s-2)} - \frac{1}{12}\frac{s}{s^2+4} - \frac{1}{6}\frac{1}{s^2+4}\right]$

$Y(s) = L^{-1}\left[\frac{13}{12(s-2)} - \frac{1}{12}\frac{s}{(s^2+4)} - \frac{2}{12(s^2+4)}\right]$

$Y(s) = \frac{1}{12} L^{-1}\left[\frac{13}{s-2} - \frac{s}{s^2+4} - \frac{2}{s^2+4}\right]$

$Y(s) = \frac{1}{12}(13e^{2t} - \cos 2t - \sin 2t)$ //

iii) $\frac{dy}{dt} - 4y = 8$. given that at $t=0, y=2$

$sY(s) - y(0) - 4Y(s) = 8/s$

$sY(s) - 2 - 4Y(s) = \frac{8}{s}$

$sY(s) - 4Y(s) = \frac{8}{s} + 2 = \frac{8-2s}{s}$

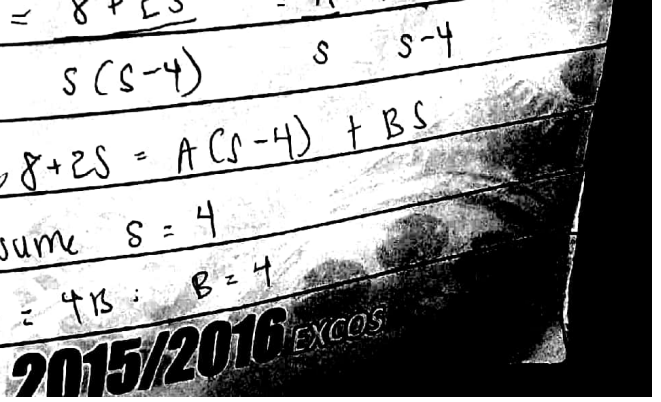
$Y(s)(s-4) = \frac{8+2s}{s}$

$Y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$8+2s = A(s-4) + Bs$

Assume $s = 4$

$16 = 4B$; $B = 4$



$S = 0$

$8 = -4A, \quad A = -2$

$$Y(s) = L^{-1}\left[\frac{-2 + 4}{s \quad s-4}\right]$$

$Y(s) = -2 + 4e^{4t}$.

$y(s) = 4e^{4t} - 2$ //

iv) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ at $t=0, y=2, y'=1$

$s^2Y(s) - sy(0) - y'(0) - 2[sY(s) - y(0)] + 5Y(s) = \frac{1}{s-2}$

$s^2Y(s) - sy(0) - y'(0) - 2sY(s) + 2y(0) + 5Y(s) = \frac{1}{s-2}$

$s^2Y(s) - 2s - 1 - 2sY(s) + 4 + 5Y(s) = \frac{1}{s-2}$

$s^2Y(s) - 2sY(s) + 5Y(s) - 2s + 3 = \frac{1}{s-2}$

$s^2Y(s) - 2sY(s) + 5Y(s) = \frac{1}{s-2} + 2s - 3$

$s^2Y(s) - 2sY(s) + 5Y(s) = \frac{1 + 2s^2 - 7s + 6}{s-2}$

$Y(s)(s^2 - 2s + 5)$

5.) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ given that $t=0,\ y=0,\ y'=2$

$s^2Y(s) - sy(0) - y'(0) - 6[sY(s) - y(0)] + 8Y(s) = \frac{1}{s-3}$

$s^2Y(s) - sy(0) - y'(0) - 6sY(s) + 6y(0) + 8Y(s) = \frac{1}{s-2}$

$s^2Y(s) - 2 - 6sY(s) + 8Y(s) = \frac{1}{s-2}$

$s^2Y(s) - 6sY(s) + 8Y(s) = \frac{1}{s-2} + 2$

$Y(s)(s^2 - 6s + 8) = \frac{1 + 2s - 4}{s-2} = \frac{2s-3}{s-2}$

$Y(s) = \frac{2s-3}{(s-2)(s-2)(s-4)}$

$\frac{2s-3}{(s-4)(s-2)^2} = \frac{A}{s-4} + \frac{B}{s-2} + \frac{C}{s-2^2}$

$2s-3 = A(s-2)^2 + B(s-4)(s-2) + C(s-4)$

Assume $s = 2$

$1 = -2C \ ;\ C = -1/2$

Assume $s = 4$

$5 = 4A \ ;\ A = 5/4$

Assume $s = 0$

$-3 = 4A + 8B - 4C$: subst A & C

$-3 = 4 \cdot 5/4 + 8B - 4 \cdot -1/2$

$-3 = 5 + 8B - 2$

$-3 = 8B + 3$

$-6 = 8B \ .\ B = -3/4$

$Y(s) = L^{-1}\left[\frac{5/4}{s-4} + \frac{-3/4}{s-2} + \frac{-1/2}{s-2^2}\right]$

$Y(s) = L^{-1}\left[\frac{5}{4(s-4)} - \frac{3}{4(s-2)} - \frac{1}{2(s-2)^2}\right]$

$Y(s) = L^{-1}\left[\frac{5}{4(s-4)} - \frac{3}{4(s-2)} - \frac{2}{4(s-2)^2}\right]$

$Y(s) = \frac{1}{4} L^{-1}\left[\frac{5}{(s-4)} - \frac{3}{(s-2)} - \frac{2}{(s-2)^2}\right]$

$Y(s) = \frac{1}{4}\left(5e^{4t} - 3e^{2t} - 2te^{2t}\right)$

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