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15/ENG06/013

ENG 381 ASSIGNMENT V

MECHANICAL ENGINEERING

1) $\frac{dy}{dx} + 3y = e^{-2t}$ given that at $t=0$, $y=2$

solution

$y' + 3y = e^{-2t}$

$L\{y'\} = sY_{(s)} - Y_{(0)}$

$L\{3y\} = 3Y_{(s)}$

$L\{e^{-2t}\} = \frac{1}{s+2}$

$\therefore \; sY_{(s)} - Y_{(0)} + 3Y_{(s)} = \frac{1}{s+2}$

$sY_{(s)} - 2 + 3Y_{(s)} = \frac{1}{s+2}$

$Y_{(s)}[s+3] = \frac{1}{s+2} + 2$

$Y_{(s)}[s+3] = \frac{2s+5}{s+2}$

$Y_{(s)} = \frac{2s+5}{s+2} \div s+3 = \frac{2s+5}{(s+2)(s+3)}$

Using partial fraction

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$2s+5 = A(s+3) + B(s+2)$

If $s = -2$

$2(-2)+5 = A(-2+3)$

$1 = A$

If $s = -3$

$2(-3)+5 = A(-3+3) + B(-3+2)$

$-1 = -B$

$B = 1$

$\therefore \; \frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$

$\therefore y(t) = L^{-1}\left\{\frac{1}{s+2}\right\} + L^{-1}\left\{\frac{1}{s+3}\right\}$

$y(t) = e^{-2t} + e^{-3t}$

2) $3\frac{dy}{dt} - 6y = \sin 2t$ at $t=0$, $y=1$

By solution

$3y' - 6y = \sin 2t$

$L\{3y'\} = 3[sY(s) - y(0)]$

$L\{-6y\} = -6Y(s)$

$L\{\sin 2t\} = \frac{2}{s^2+2^2}$

$\therefore 3[sY(s) - y(0)] - 6Y(s) = \frac{2}{s^2+2^2}$

$3sY(s) - 3y(0) - 6Y(s) = \frac{2}{s^2+2^2}$

$3sY(s) - 6Y(s) - 3(1) = \frac{2}{s^2+2^2}$

$3sY(s) - 6Y(s) = \frac{2}{(s^2+2)^2} + 3$

$Y(s)[3s-6] = \cancel{\frac{3s^2+14}{s^2+4}}\quad \frac{2}{s^2+4} + 3 = \frac{3s^2+14}{s^2+4}$

$\cancel{Y(s) = \frac{3s^2+14}{(s+4)} \div (3s-6) = \frac{3s^2+14}{(s+4)(3s-6)}}$

$Y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{3s^2+14}{(3s-6)(s^2+4)}$

Using partial fraction

$Y(s) = \frac{A}{3s-6} + \frac{Bs+C}{s^2+4}$

$3s^2+14 = A(s^2+4) + (Bs+C)(3s-6)$

If $s=2$

$3(2)^2 + 14 = A(2^2+4) + (B(2)+C)(3(2)-6)$

$12+14 = 8A$

$A = \frac{26}{8} = \frac{13}{4}$

Comparing coefficients

$3s^2 + 14 = As^2 + 4A + 3Bs^2 - 6Bs + 3Cs - 6C$

$3s^2 = As^2 + 3Bs^2$

$3 = A + 3B$ — ①

$0 = -6Bs + 3Cs$ —

$0 = -6B + 3C$ — ②

$14 = 4A - 6C$ — ③

from equ ①

$3 = A + 3B$

$3 - A = 3B$

$\frac{3 - A}{3} = B$; $3B = 3 - \frac{13}{4} = -\frac{1}{4}$

$B = -\frac{1}{4} \div 3 = -\frac{1}{4} \times \frac{1}{3} = -\frac{1}{12}$

from equ ②

$3C = 6B$

$C = \frac{6B}{3} = 2B = 2 \times -\frac{1}{12} = -\frac{1}{6}$

$\therefore \frac{3s^2 + 14}{(3s-6)(s^2+4)} = \frac{13}{4(3s-6} - \frac{s-1}{12(s^2+4)6} = \frac{13}{4(3s-6)} - \frac{s-1}{72(s^2+4)} = \frac{13}{12(s-2)} - \frac{s-1}{72(s^2+4)}$

$y(t) = \frac{13}{2} e^{2t} - \frac{1}{72} \cos 2t$

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3) $\frac{dy}{dt} - 4y = 8$ at $t=0,\ y=2$

$y' - 4y = 8$

$L\{y'\} = [sY_{(s)} - Y_{(0)}]$

$L\{-4y\} = -4Y_{(s)}$

$L\{8\} = \frac{8}{s}$

$\therefore\ sY_{(s)} - Y_{(0)} - 4Y_{(s)} = \frac{8}{s}$

$sY_{(s)} - 2 - 4Y_{(s)} = \frac{8}{s}$

$Y_{(s)}[s-4] = \frac{8}{s} + 2$

$Y_{(s)}[s-4] = \frac{2s+8}{s}$

$Y_{(s)} = \frac{2s+8}{s} \div [s-4] = \frac{2s+8}{s(s-4)}$

using partial fraction

$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$2s+8 = A(s-4) + B(s)$

if $s=4$

$2(4)+8 = A(4-4) + B(4)$

$16 = 4B$

$B = 4$

if $s = 0$

$2(0)+8 = A(0-4) + B(0)$

$8 = -4A$

$A = -2$

$\therefore\ \frac{2s+8}{s(s-4)} = \frac{-2}{s} + \frac{4}{s-4}$

$Y_{(t)} = L^{-1}\left\{ -2/s + 4/(s-4) \right\}$

$Y_{(t)} = -2 + 4e^{4t}$

4) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0$, $y=0$, $y'=2$

Solution

$y'' - 6y' + 8y = e^{3t}$

$L\{y''\} = s^2 Y_{(s)} - sY_{(0)} - y'_{(0)}$

$L\{-6y'\} = -6[sY_{(s)} - Y_{(0)}]$

$L\{8y\} = 8Y_{(s)}$

$L\{e^{3t}\} = \frac{1}{s-3}$

$\therefore s^2 Y_{(s)} - sY_{(0)} - y'_{(0)} - 6[sY_{(s)} - Y_{(0)}] + 8Y_{(s)} = \frac{1}{s-3}$

$s^2 Y_{(s)} - s(0) - 2 - 6sY_{(s)} - 0 + 8Y_{(s)} = \frac{1}{s-3}$

$s^2 Y_{(s)} - 6sY_{(s)} + 8Y_{(s)} = \frac{1}{s-3} + 2$

$Y_{(s)}[s^2 - 6s + 8] = \frac{1}{s-3} + 2 = \frac{1 + 2(s-3)}{s-3} = \frac{1 + 2s - 6}{s-3} = \frac{2s-5}{s-3}$

$Y_{(s)} = \frac{2s-5}{s-3} \div [s^2 - 6s + 8] = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{s-3} + \frac{Bs+C}{(s^2-6s+8)}$

$2s - 5 = A(s^2 - 6s + 8) + (Bs + C)(s-3)$

$2s - 5 = As^2 - 6As + 8A + Bs^2 - 3Bs + Cs - 3C$

Comparing Coefficients

$A + B = 0$ —— ①

$-6A - 3B + C = 2$ —— ②

$8A - 3C = -5$ —— ③

$B = -A$ from ①

$-6A - 3(-A) + C = 2$

$-6A + 3A + C = 2$

$-3A + C = 2$ —— ④ $\times -3$

~~$C = 2 + 3A$~~

$8A - 3C = -5$.

$8A - 3(2 + 3A) = 2$ $\quad 9A - 3C = -6$ —— ⑤

$8A - 6 - 9A = 2$ $\quad$ equ ⑤ - ④

$A = -1$

$\therefore A = -B$ or $B = -A$

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$B = -(-1) = 1$

from ③

$8(-1) - 3c = -5$

$-8 + 5 = 3c$

$-3 = 3c$

$c = -1$

$$\therefore \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{1}{s-3} + \frac{s-1}{(s^2-6s+8)}$$

$$\frac{s-1}{s^2-6s+8} = \frac{A}{s-2} + \frac{B}{s-4}$$

$s - 1 = A(s-4) + B(s-2)$

If $s = 2$

$2 - 1 = A(-2)$

$1 = -2A$

$A = -1/2$

If $s = 4$

$4 - 1 = B(4-2)$

$3 = 2B$

$B = 3/2$

$$\frac{s-1}{(s-2)(s-4)} = \frac{-1/2}{s-2} + \frac{3/2}{s+4}$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{-1}{s-3} - \frac{1/2}{s-2} + \frac{3/2}{s-4}\right\}$$

$y = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$

$y = \frac{1}{2}\left[2e^{3t} + e^{2t} - 3e^{4t}\right]$

5) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ at $t=0, y=2, y'=1$

solution

$L\{y''\} = s^2Y_{(s)} - sY_{(0)} \cancel{+} - y'_{(0)}$

$L\{-2y'\} = -2[sY_{(s)} - Y_{(0)}]$

$L\{5y\} = 5Y_{(s)}$

$L\{e^{2t}\} = \frac{1}{s-2}$

$s^2Y_{(s)} - sY_{(0)} - y'_{(0)} - 2[sY_{(s)} - Y_{(0)}] + 5Y_{(s)} = \frac{1}{s-2}$

$s^2Y_{(s)} - 2sY_{(s)} + 5Y_{(s)} - s(2) - 1 + 2(2) = \frac{1}{s-2}$

$s^2Y_{(s)} - 2sY_{(s)} + 5Y_{(s)} - 2s - 1 + 4 = \frac{1}{s-2}$

$Y_{(s)}[s^2 - 2s + 5] = \frac{1}{s-2} + 2s - 3$

$Y_{(s)}[s^2 - 2s + 5] = \frac{(2s-3)(s-2)}{s-2}$

$Y_{(s)} = \frac{(2s-3)(s-2)}{(s-2)} \div [s^2 - 2s + 5] = \frac{1 + 2s^2 - 5 + 6}{(s-2)(s^2-2s+5)}$

$\frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{A}{(s-2)} + \frac{Bs + C}{(s^2-2s+5)}$

$2s^2 - 7s + 7 = A(s^2-2s+5) + (Bs+C)(s-2)$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$, (comparing coefficients)

$A + B = 2$ — (1)

$-2A - 2B + C = -7$ — (2)

$5A - 2C = 7$ — (3)

from (1)

$B = 2 - A$

$-2A - 2(2-A) + C = -7$

$-2A + 4 + 2A + C = -7$

$C = 3$

from (3)

$5A - 2(-3) = 7$

$5A = 7 - 6$

$5A = 1$

$A = \frac{1}{5}$

$\therefore A + B = 2$

$\frac{1}{5} + B = 2$

$B = 2 - \frac{1}{5} = \frac{9}{5}$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{1/5}{(s-2)} + \frac{\frac{9}{5}s - 3}{(s^2 - 2s + 5)}$$

$$= \frac{1/5}{s-2} + \frac{\frac{9}{5}s}{(s^2 - 2s + 5)} - \frac{3}{(s^2 - 2s + 5)}$$

$$= \frac{1}{5(s-2)} + \frac{9}{5}\left[\frac{(s-1)}{(s-1)^2 + 2^2} - \frac{1 \times 2/2}{(s-1)^2 + 2^2}\right] - \frac{3}{2}\left[\frac{2}{(s-1) + 2^2}\right]$$

$$= L^{-1}\left\{\frac{1/5}{(s-2)} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2 + 2^2} + \frac{1}{2}\left(\frac{2}{(s-1)^2 + 2^2}\right)\right] - \frac{3}{2}\left(\frac{2}{(s-1)^2 + 2}\right)\right.$$

$$= \frac{1}{5}e^{2t} + \frac{9}{5}\left[e^t \cos 2t + \frac{1}{2} e^t \sin 2t\right] - \frac{3}{2}\left(e^t \sin 2t\right)$$