15/ENG05/017

MECHATRONICS

OBIEKWE ARINZE HENRY

ENG 381

i) $dy/dt + 5y = e^{-2t}$

$y(t) + 5y(t) = e^{-2t}$

$L(y'(t)) = sy(s) - y(s)$

$L[y(t)] = Y(s) + L^{-1}(e^{-2t}) = 1/s+2$

$sy(s) - Y(s) + 5y(s) = 1/s+2$

$(s-3)\,y(s) - 2 = 1/s+2$

$(s-3)\,y(s) = 1/s+2 + 2 = \frac{1+2s+4}{s+2} = 2s+5/s+2$

$Y(s) = \frac{2s+5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$

$2(-2)+5 = A(-2-3) \Rightarrow A = 1/-5$

$2(3)+5 = B(3+2) \Rightarrow B = 11/5$

$y(s) = \frac{-1}{5(s+2)} + \frac{11}{5(s-3)} = \frac{-1}{5}e^{-2t} + \frac{11}{5}e^{3t}$

ii) $3\,dy/dt - 6y = \sin 2t = 3y'(t) - 6y(t)$

$L[y'(t)] = sy(s) - y(0)$

$L[y(t)] = y(s)$

$L[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$

$3s + Y(s) - 3y(0) - 6y(s) = 2/s^2+4$

$y(s)[3s-6] = \frac{2}{s^2+4} + 3 = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$

$y(s) = \frac{3s^2+14}{(3s-6)(s^2+4)}$

$= \frac{A}{3s-6} + \frac{B}{s^2+4}$

$3s^2+14 = A(s^2+4) + B(3s-6)$

$4A - 6B = 14$

$-6B = 14 - 12$

$B = -1/3$

$$Y(s) = \frac{3}{[s][s-2]} - \frac{1}{3(s^2+4)}$$

$$= e^{2t} - \frac{1}{6}\sin 2t$$

iii) $dy/dt - 4y = 8$

$y'(t) - 4y(t) = 8$

$y'(t) - 4y(t) = 8$

$sY(s) - Y(0) - 4y(s) = 8/s$

$(s-4)[Y(s)] = 8/s + 2 = \frac{8+2s}{s}$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$2s + 8 = A(s-4) + B(s)$

$2(0) + 8 = A(0-4) \Rightarrow A = -2$

$2(4) + 8 = A(4-4) + B(4)$

$B = 4$

$$L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right] = -2 + 4e^{4t}.$$

iv) $\frac{d^2y}{dt^2} - 2dy/dt + 5y = e^{2t}$

$y'(t) - 2y'(t) + 5y(t) = e^{2t}$

$L[y''(t)] = s^2 y(s) - sy(0) - y'(0)$

$L[y'(t)] = sy(s) - y(s)$

$L[y(t)] = Y(s)$

$s^2y(s) - sy(s) - y'(s) - 2sy(s) + 2y(s) + 5y(s) = 1/s-2$

$(s^2 - 2s + 5)y(s) + (2-s)y_0 - y'(0) = 1/s-2$

$(s^2 - 2s + 5)y(s) = 1/s-2 - [2-s)2 + 1$

$$= \frac{1}{s-2} + 2s - 4 + 1 = \frac{1}{s-2} + 2s - 3$$

$$= \frac{1 + (2s-3)(s-2)}{} = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$$

$$Y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)}$$

$$2s^2 - 7s + 7 = \frac{A}{s-2} + \frac{B}{s^2-2s+5}$$

$$2s^2 - 2s + 7 = A(s^2 - 2s + 5) + B(s-2)$$

$A = 2$

$$-2A + B = -7$$

$$B = -7 + 4 = -3$$

$$= \frac{2}{s-3} \quad \frac{-3}{s-2s+5}$$

$$= 2e^{2t} - 7/3 + \sin 2t$$

4) $d^2y/dt^2 - 6\,dy/dt + 8y = e^{3t}$

$$s^2 y(s) - sy(0) - y'(0) - 6[y(s) + 6y(0) + sy(s)] = \frac{1}{s-3}$$

$$[s^2 - 6s + 5]\,y(s) + (6-s)\,y(0) - y'(0) = \frac{1}{s-3}$$

$$[s^2 - 6s + 8]\,y(s) = \frac{1}{s-3} + 2 = \frac{1 + 2s - 6}{s-3} = \frac{2s-5}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$2s - 5 = A(s-2)(s-4) + B(s-3)(s-4) + [(s-3)(s-2)]$$

$$2s - 5 = A[s^2 - 6s + 8] + B[s^2 - 7s + 12] + C[s^2 - 5s + 6]$$

$$2[3] - 5 = A[3-2][3-4] \rightarrow A = -1$$

$$2[4] - 5 = C((4-3)(4-2) \Rightarrow C = 3/2$$

$$-6A - 7B - 5(.7$$

$$-6[-1] - 7B - 5(3/2) = 2$$

$$-7B = 2 + 3/2 - 6 = \frac{4 + 15 - 12}{2} = 7/2$$

$$L^{-1}\left[\frac{-1}{s-3} \quad \frac{-1}{2(s-2)} + \frac{3}{2(s-4)}\right]$$

$$= -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$