

ENGG CHEMISTRY KJZITO  
 MECHATRONICS  
 15/ENG08008  
 ENGG 381 ASSIGNMENT

Q

$$dy/dt + 3y = e^{-2t}$$

$$y(t) + 3y(t) = e^{-2t}$$

$$L[y'(t)] = sy(s) - y(0)$$

$$L[y(t)] = y(s), L^{-1}[e^{-2t}] = \frac{1}{s+2}$$

$$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$$

$$(s-3)y(s) - 2 = \frac{1}{s+2}$$

$$(s-3)y(s) - 2 = \frac{1}{s+2}$$

$$(s-3)y(s) = \frac{1}{s+2} + 2 = \frac{1+2s+4}{s+2} = \frac{2s+5}{s+2}$$

$$y(s) = \frac{2s+5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$$

$$2(-2)+5 = A(-2-3) \Rightarrow A = \frac{1}{-5}$$

$$2(3)+5 = B(3+2) \Rightarrow B = \frac{11}{5}$$

$$y(s) = \frac{-1}{5(s+2)} + \frac{11}{5(s-3)} = \frac{-1}{5} e^{-2t} + \frac{11}{5} e^{3t}$$

Q

$$3dy/dt - 6y = \sin 2t = 3y'(t) - 6y(t)$$

$$L[y'(t)] = sy(s) - y(0)$$

$$L[y(t)] = y(s)$$

$$L[\sin 2t] = \frac{2}{s^2+4} = \frac{2}{s^2+4}$$

$$3s - 1y(s) - 3y(0) - 6y(s) = \frac{2}{s^2+4}$$

$$y(s)[3s-6] = \frac{2}{s^2+4} + 3 = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$$

$$y(s)[3s-6] = \frac{2}{s^2+4} + 3 = \frac{2+3s^2+12}{s^2+4}$$

$$y(s) = \frac{3s^2+14}{(3s-6)(s^2+4)}$$

$$= \frac{A}{3s-6} + \frac{B}{s^2+4}$$

$$3s^2+14 = A(s^2+4) + B(3s-6)$$

$$4A - 6B = 14$$

$$-6B = 14 - 12$$

$$B = -\frac{1}{3}$$

$$y(s) = \frac{3}{[3][s-2]} - \frac{1}{3(s^2+4)}$$
$$= e^{2t} - \frac{1}{6} \sin 2t$$

(ii)  $\frac{dy}{dt} - 4y = 8$

$$y'(t) - 4y(t) = 8$$

$$y'(t) - 4y(t) = 8$$

$$sY(s) - y(0) - 4Y(s) = \frac{8}{s}$$

$$(s-4)Y(s) = \frac{8}{s} + 2 = \frac{8+2s}{s}$$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s+8 = A(s-4) + B(s)$$

$$2(0)+8 = A(0-4) \Rightarrow A = -2$$

$$2(4)+B = A(4-4) + B(4)$$

$$B = 4$$

$$L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right] = -2 + 4e^{4t}$$

(iv)  $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$$y''(t) - 2y'(t) + 5y(t) = e^{2t}$$

$$L[y''(t)] = s^2Y(s) - sy(0) - y'(0)$$

$$L[y'(t)] = sY(s) - y(0)$$

$$L[y(t)] = Y(s)$$

$$s^2Y(s) - sy(0) - y'(0) - 2[sY(s) - y(0)] + 5Y(s) = \frac{1}{s-2}$$

$$(s^2 - 2s + 5)Y(s) + (2-s)y_0 - y'(0) = \frac{1}{s-2}$$

$$(s^2 - 2s + 5)Y(s) = \frac{1}{s-2} - [2-s]y_0 + 1$$

$$= \frac{1}{s-2} + 2s - 4 + 1 = \frac{1}{s-2} + 2s - 3$$

$$= \frac{1 + (2s-3)(s-2)}{(s-2)} = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$$

$$(s-2)(s^2-2s+5)$$

$$s^2 - 7s + 7 = \frac{A}{s-2} + \frac{B}{s^2-2s+5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$$

$$A = 2$$

$$-2A + B = -7$$

$$B = -7 + 4 = -3$$

$$= \frac{2}{s-2} - \frac{3}{s^2-2s+5}$$

$$= 2e^{2t} - \frac{7}{3}t \sin 2t$$

v)  $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$

$$s^2 y(s) - sy(0) - y'(0) - 6sy(s) + 6y(0) + 8y(s) = \frac{1}{s-3}$$

$$[s^2 - 6s + 8]y(s) + (6-3)y(0) - y'(0) = \frac{1}{s-3}$$

$$[s^2 - 6s + 8]y(s) = \frac{1}{s-3} + 2 = \frac{1-2s-6}{s-3} = \frac{2s-5}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$2s-5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-3)(s-2)$$

$$2s-5 = A[s^2-6s+8] + B[s^2-7s+12] + C[s^2-5s+6]$$

$$2[8]-5 = A[3-2][3-4] \Rightarrow A = -1$$

$$2[4]-5 = C[4-3][4-2] \Rightarrow C = \frac{3}{2}$$

$$-6A - 7B - 5C = 7$$

$$-6[-1] - 7B - 5[\frac{3}{2}] = 7$$

$$-7B = 7 + \frac{15}{2} - 6 = \frac{4+15-12}{2} = \frac{7}{2}$$

$$L^{-1} \left[ \frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)} \right]$$

$$= -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$