15/ENG07/016

Petroleum Engineering

i $\frac{dy}{dt} + 3y = e^{-2t}$, given that at $t=0, y=2$

$L\{dy/dt\} = sY(s) - y(0)$

$L\{y\} = Y(s)$

$L\{e^{-2t}\} = \frac{1}{s+2}$

$sY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$

$\cancel{s}Y(s)(s+3) - 2 = \frac{1}{s+2}$

$Y(s)(s+3) = \frac{1}{s+2} + 2$

$Y(s)(s+3) = \frac{1+2s+4}{s+2}$

$Y(s) = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

~~When s = -2,~~ $2s+5 = A(s+3) + B(s+2)$

~~1 =~~ When $s=-2$,

$1 = A$

When $s = -3$,

$-1 = -B$

$\therefore B = 1$

$L^{-1}(Y(s)) = L^{-1}\left[\frac{1}{s+2} + \frac{1}{s+3}\right]$

$y(t) = e^{-2t} + e^{-3t}$.

ii $3\,dy/dt - 6y = \sin 2t$, given that $y(0) = 1$

$L\{dy/dt\} = sY(s) - y(0)$

$L(y) = Y(s)$

$L(\sin 2t) = \frac{2}{s^2+4}$

$3sY(s) - 3y(0) - 6Y(s) = \frac{2}{s^2+4}$

$Y(s)(3s-6) - 3 = \frac{2}{s^2+4}$

$Y(s)(3s-6) = \frac{2}{s^2+4} + 3$

$Y(s)(3s-6) = \frac{2+3s^2+12}{s^2+4}$

$$Y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)}$$

$$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{As+B}{s^2+4} + \frac{C}{3s-6}$$

$$3s^2+14 = (As+B)(3s-6) + C(s^2+4)$$

$$3s^2+14 = 3As^2 - 6As + 3Bs - 6B + Cs^2 + 4C$$

$$3A + C = 3 \longrightarrow (i)$$

$$-6A + 3B = 0 \longrightarrow (ii)$$

$$-6B + 4C = 14 \longrightarrow (iii)$$

From (i), $C = 3 - 3A$

From (iii), $-6B + 12 - 12A = 14$

$$-12A - 6B = 2 \longrightarrow (iv)$$

Eqn. (ii) $\times$ 2 $\quad -12A + 6B = 0 \longrightarrow (v)$

Solving (iv) and (v) simultaneously:

$$-24A = 2$$

$$\therefore A = -\frac{1}{12}$$

From (i), $-\frac{1}{4} + C = 3$

$$\therefore C = \frac{13}{4}$$

From (iii), $-6B + \frac{52}{4} = 14$

$$\therefore B = -\frac{1}{6}.$$

$$\mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}\left[\frac{-\frac{1}{12}s - \frac{1}{6}}{s^2+4} + \frac{13/4}{3s-6}\right]$$

$$= \frac{-1/12\, s}{s^2+4} + \frac{-1/6}{s^2+4} + \frac{13/4}{3s-6}$$

$$= -\frac{1}{12}\left[\frac{s}{s^2+2^2}\right] - \frac{1}{12}\left[\frac{2}{s^2+2^2}\right] + \frac{13}{12}\left[\frac{1}{s-2}\right]$$

$$\therefore y(t) = -\frac{1}{12}\cos 2t - \frac{1}{12}\sin 2t + \frac{13}{12}e^{2t}.$$

ii $\quad dy/dt - 4y = 8$, given that $y(0) = 2$, ~~$y'(0) =$~~

$$\mathcal{L}\{dy/dt\} = sY(s) - y(0)$$

$$\mathcal{L}\{y\} = \cancel{Y(s)}\ Y(s)$$

$$\mathcal{L}\{8\} = \frac{8}{s}$$

$$sY(s) - y(0) - 4Y(s) = 8/s$$

$$Y(s)(s-4) - 2 = 8/s$$

$$Y(s)(s-4) = 8/s + 2$$

$$Y(s)(s-4) = \frac{2s+8}{s}$$

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$$Y(s) = \frac{2s+8}{s(s-4)}$$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s+8 = A(s-4) + Bs$$

When $s = 4$,

$$16 = \cancel{Bs}\ 4B$$

$$\therefore B = 4.$$

$$2s+8 = As - 4A + 4s$$

$$A + 4 = 2$$

$$\therefore A = -2$$

$$L^{-1}\{Y(s)\} = L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right]$$

$$\therefore y(t) = -2 + 4e^{4t}.$$

iv $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$, given that $y(0) = 2$, $y'(0) = 1$

$$L[d^2y/dt^2] = s^2Y(s) - sy(0) - y'(0)$$

$$L[dy/dt] = sY(s) - y(0)$$

$$L(5y) = Y(s)$$

$$L\{e^{2t}\} = \frac{1}{s-2}$$

$$s^2Y(s) - sy(0) - y'(0) - 2sY(s) + 2y(0) + 5Y(s) = \frac{1}{s-2}$$

$$Y(s)(s^2 - 2s + 5) \cancel{-sy(0) + y(0)(s+2)} - 2s - 1 + 4 = \frac{1}{s-2}$$

$$Y(s)(s^2 - 2s + 5) - 2s + 3$$

$$Y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 3$$

$$Y(s)(s^2 - 2s + 5) = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$$

$$Y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)}$$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs+C}{s^2-2s+5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + Bs+C(s-2)$$

When $s = 2$,

$$8 - 14 + 7 = A(4 - 4 + 5) + 0$$

$$1 = 5A$$

$$\therefore A = 1/5$$

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$2s^2-7s+7 = As^2-2As+5A+Bs^2-2Bs+Cs-2C$

$2s^2-7s+7 = \frac{1}{5}s^2-\frac{2}{5}s+1+Bs^2-2Bs+Cs-2C$

$\frac{1}{5}+B=2$

$B = 2-\frac{1}{5}$

$\therefore B = 9/5.$

$1-2C=7$

$2C=-6$

$\therefore C=-3.$

$\therefore L^{-1}\{Y(s)\} = L^{-1}\left[\frac{1/5}{s-2}+\frac{\frac{9}{5}s \cancel{+} -3}{s^2-2s+5}\right]$

But $s^2-2s+5 = s^2-2s+1+4 = (s^2-2s+1)+2^2$

$s^2-2s+1 = s^2-s-s+1$

$= s(s-1)-1(s-1)$

$= (s-1)(s-1) = (s-1)^2$

$\therefore s^2-2s+5 = (s-1)^2+2^2$

$L^{-1}\{Y(s)\} = L^{-1}\left[\frac{1/5}{s-2}+\frac{\frac{9}{5}s-3}{(s-1)^2+2^2}\right]$

$\frac{1}{5}\left(\frac{1}{s-2}\right)+\frac{9/5\, s}{(s-1)^2+2^2} - \frac{3}{(s-1)^2+2^2}$

$\cancel{y(t) = \frac{1}{5}e^{2t} + \frac{9e^{t}\cos 2t}{5} - 3t\sin 2t}$

$\therefore y(t) = \frac{1}{5}e^{2t}+\frac{9}{5}e^{t}\cos 2t-\frac{3}{2}e^{t}\sin 2t.$

√ $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$, given that $y(0) = 0$, $y'(0) = 2$

$\mathcal{L}\{y''\} = s^2Y(s) - sy(0) - y'(0)$

$\mathcal{L}\{y'\} = sY(s) - y(0)$

$\mathcal{L}\{y\} = Y(s)$

$\mathcal{L}\{e^{3t}\} = 1/s-3$

$s^2Y(s) - sy(0) - y'(0) - 6sY(s) + 6y(0) + 8Y(s) = \frac{1}{s-3}$

$Y(s)\{s^2 - 6s + 8\} - 2 = \frac{1}{s-3}$

$Y(s)[s^2 - 6s + 8] = \frac{2s-5}{s-3}$

$Y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$

$2s - 5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-3)(s-2)$

Let $s = 2$,

$\cancel{2s-5} = 2B = -1 \quad \therefore B = -1/2$

Let $s = 4$,

$2C = 3 \quad \therefore C = 3/2$

$2s - 5 = As^2 - 6As + 8A + Bs^2 - 7Bs + 12B + Cs^2 - 5Cs + 6C$

$8A + 12B + 6C = -5$

$8A = -5 + 6 - 9$

$\therefore A = -1.$

$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{-1}{s-3} - \frac{1}{2}\cdot\frac{1}{s-2} + \frac{3}{2}\cdot\frac{1}{s-4}\right\}$

$\therefore y(t) = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}.$