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15/ENG03/010

$(s^2y(s) - sy(s) - y'(0)) - 2(sy(s) - y(s))$

$+ sy(s) = \frac{1}{s-2}$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + sy(s) = \frac{1}{s-2}$

$y(s)[s^2 - 2s + s] - \frac{1}{s-2} + \frac{2s}{1} - \frac{3}{1} =$

$\frac{1 + 2s(s-2) - 3(s-2)}{s-2}$

$y(s)[s^2 - 2s + s] = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$

$y(s)[s^2 - 2s + s] = \frac{2s^2 + 7s + 7}{s-2}$

$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + s)}$

$\Rightarrow \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + s}$

$A: \left.\frac{2s^2 - 7s + 7}{s^2 - 2s + s}\right]_{s=-2} =$

$= \frac{2(2)^2 - 7(2) + 7}{2^2 - 2(2) + s} = \frac{1}{s}$

$\Rightarrow 2s^2 - 7s + 7 = A(s^2 - 2s + s) + (Bs + C)$
$(s-2)$

$\Rightarrow 2s^2 - 7s + 7 = As^2 - 2As + sA +$
$Bs - 2Bs + Cs - 2C$

$2 = A + B$

$2 = \frac{1}{s} + B$

$B = \frac{9}{s}$

$7 = sA - 2C$

$7 = s(\frac{1}{s}) - 2C$

$7 - 1 = 2C$

$C = -3$

$y(s) = \frac{1}{s} \div s-2 + \frac{\frac{9s}{s} - 3}{s^2 - 2s + s} - \frac{1}{s}$

$\times \frac{1}{s-2} + \frac{\frac{9s}{s}}{(s+1)^2 + 4} - \frac{3}{(s+1)^2}$

$y(s) = \frac{1}{s} \frac{1}{s-2} + \frac{9}{s} \frac{s-1+1}{(s+1)^2 + 4} - \frac{3}{(s+1)^2 + 2^2}$

$y(s) = \frac{1}{s} \cdot \frac{1}{s-2} + \frac{9}{s} \frac{(s+1)}{(s+1)^2 + 4}$

$- \frac{3}{(s+1)^2 + 2^2}$

$y(s) = \frac{1}{s} \cdot \frac{1}{s-2} + \frac{9}{s} \frac{(s+1)}{(s+1)^2 + 2^2}$

$- \frac{1}{(s+1)^2 + 4} - \frac{3}{(s+1)^2 + 2^2}$

$y(s) = \frac{1}{s} \cdot \frac{1}{s-2} + \frac{9}{s} \frac{s+1}{(s+1)^2 + 2} -$

$\frac{4}{(s+1)^2 + 2^2} \cdot \frac{2}{2}$

$y(t) = \frac{1}{s}[e^{2t} + \frac{9}{s}e^{-t}\cos 2t -$
$2e^{-t}\sin 2t$

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1. $\frac{dy}{dt} + 3y = e^{-2t}$ given at $t=0$; $y=2$

$y' + 3y' = e^{-2t}$

$s\,y(s) - y(s) + 3y(s) = \frac{1}{s+2}$

$s\,y(s) - 2 + 3y(s) = \frac{1}{s+2}$

$y(s)[s+3] = \frac{1}{s+2} + \frac{2}{1} \Rightarrow \frac{1+2(s+2)}{s+2}$

$y(s)[s+3] = \frac{1+2s+4}{s+2}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$A : \left.\frac{1+2s+4}{s+3}\right]_{s=-2} = \frac{1+2(-2)+4}{-2+3} = 1$

$B : \left.\frac{1+2s+4}{s+3}\right]_{s=-3} = \frac{1+2(-3)+4}{-3+2} = 1$

$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = e^{-2t} + e^{-3t}$

2 $3\frac{dy}{dt} - 6y = \sin 2t$ at $t=0$; $y=1$; $y(s)=1$

$3y' - 6y = \sin 2t$

$3(s\,y(s) - y(0)) - 6y(s) = \frac{2}{s+4}$

$3s\,y(s) - 3y(0) - 6y(s) = \frac{2}{s+4}$

$y(s)[3s-6] = \frac{2}{s^2+4} + \frac{3}{1} \Rightarrow \frac{2+3(s^2+4)}{s^2+4}$

$y(s)[3s-6] = \frac{2+3s^2+12}{s^2+4} \Rightarrow \frac{3s^2+14}{(s^2+4)}$

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$$y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+B}{(s^2+4)} + \frac{C}{(3s-6)}$$

$$C: \left.\frac{3s^2+14}{s^2+4}\right]_{s=2} = \frac{3(2)^2+14}{2^2+4} = \frac{13}{4}$$

$$3s^2+14 = (A+B)(3s-6) + C(s^2+4)$$

$$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C.$$

Comparing Co-efficient.

$$3 = 3B + C$$

$$3 = 3B + \frac{13}{4}$$

$$3A - 6B = 0$$

$$3A = 6B$$

$$3A = 6 \times -\frac{1}{12}$$

$$A = -\frac{1}{6}$$

$$y(s) = \frac{-\frac{1}{6} - \frac{1}{12}s}{s^2+4} + \frac{\frac{13}{4}}{3s-6}$$

$$\frac{-\frac{1}{6}}{s^2+4} - \frac{\frac{1}{12}s}{s^2+4} + \frac{\frac{13}{4}}{3s-6}$$

$$= -\frac{1}{6}\cdot\frac{1}{s^2+4^2} - \frac{1}{12}\left[\frac{s}{s^2+2^2}\right] + \frac{13}{13}\left[\frac{1}{s-2}\right]$$

$$y(t) = -\frac{1}{12}\sin 2t - \frac{1}{12}\cos 2t + \frac{13}{12}e^{2t}$$

$$y(t) = \frac{1}{12}\left[-\sin 2t - \cos 2t + 13e^{2t}\right]$$

$$y(t) = \frac{1}{12}\left[13e^{2t} - \cos 2t - \sin 2t\right]$$

B

$$\frac{dy}{dt} - 4y = 8 \text{ given that } t=0,\ y=2,\ y(0)=2$$

$$y' - 4y = 8$$

$$sy(s) - y(0) - 4y(s) = \frac{8}{s}$$

$$sy(s) - 2 - 4y(s) = \frac{8}{s}$$

$$y(s)[s-4] = \frac{8}{s} + \frac{2}{1} = \frac{8+2s}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$A: \left.\frac{8+2s}{s-4}\right]_{s=0} = \frac{8}{-4} = -2$$

$$B: \left.\frac{8+2s}{s}\right]_{s=4} = \frac{8+2(4)}{4} = 4$$

$$y(s) = \frac{-2}{s} + \frac{4}{s-4}$$

$$y(t) = -2 + 4e^{4t}.$$

B4

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2t}$$

at $t=0$; $y=2$, $y'=1$, $y(0)=2$

$$y(0) = 1$$

$$y'' - 2y' + 5y = e^{2t}$$

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4 cont'd.

$$y(t) = \frac{1}{5}\left[e^{2t} + 9e^{-t}\cos 2t - 10e^{-t}\sin 2t\right].$$

5. $\frac{d^2y}{dx^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0$ $y=0$ $y'=2$

$$y'' - 6y' + 8y = e^{3t}$$

$$s^2y(s) - sy(s) - y'(0) - 6[sy(s) - y(0)] + 8y(s) = \frac{1}{s-3}.$$

$$s^2y(s) - 2 - 6sy(s) + 8y(s) = \frac{1}{s-3}$$

$$y(s)[s^2 - 6s + 8] = \frac{1}{s-3} + \frac{2}{1} \Rightarrow \frac{1+2(s-3)}{s-3}$$

$$y(s)[s^2 - 6s + 8] = \frac{1+2s-6}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{2s-5}{(s-3)(s-2)(s-4)} \Rightarrow \frac{A}{(s-3)} + \frac{B}{(s-2)} - \frac{C}{(s-4)}$$

$$A: \left.\frac{2s-5}{(s-2)(s-4)}\right|_{s=3} = \frac{2(3)-5}{(3-2)(3-4)} = -1$$

$$B: \left.\frac{2s-5}{(s-2)(s-4)}\right|_{s=2} = \frac{2(2)-5}{(2-3)(2-4)} = -\frac{1}{2}$$

$$C: \left.\frac{2s-5}{(s-3)(s-2)}\right|_{s=4} = \frac{2(4)-5}{(4-3)(4-3)} = \frac{3}{2}$$

$$y(s) = \frac{-1}{s-3} - \frac{1}{2} = \frac{1}{s-2} + \frac{3}{2}\cdot\frac{1}{s-4}$$

$$y(t) = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$

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