UYAEBU, Ebube
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Elect/Elect.

1) $\frac{dy}{dt} + 3y = e^{-2t}$ given at $t = 0$ $y = 2$

$y(s) = 2$

$y' + 3y'' = e^{-2t}$

$Sy(s) - y(s) + 3y(s) = \frac{1}{s+2}$

$Sy(s) - 2 + 3y(s) = \frac{1}{s+2}$

$y(s)[s+3] = \frac{1}{s+2} + \frac{2}{1} = \frac{1 + 2(s+2)}{s+2} = \frac{1 + 2s + 4}{s+2}$

$y(s) = \frac{1 + 2s + 4}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$A: \left.\frac{1 + 2s + 4}{s+3}\right|_{s=-2} = \frac{1 + 2(-2) + 4}{-2 + 3} = 1$

$B: \left.\frac{1 + 2s + 4}{s+3}\right|_{s=-3} = \frac{1 + 2(-3) + 4}{-3 + 2} = 1$

$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = e^{-2t} + e^{-3t}$.

2) $\frac{3dy}{dt} - 6y = \sin 2t$ at $= 0$ $y = 1$

$y(s) = 1$

$3y^{(1)} - 6y = \sin 2t$.

$3(Sy(s) - y(s)) - 6y(s) = \frac{2}{s+4}$

$3Sy(s) - 3y(0) - 6y(s) = \frac{2}{s+4}$

$y(s)[3s - 6] = \frac{2}{s^2+4} + \frac{3}{1} = \frac{2 + 3(s^2+4)}{s^2+4}$

$y(s)[3s - 6] = \frac{2}{s^2+4} + \frac{3}{1} = \frac{2 + 3(s^2+4)}{s^2+4}$

$y(s)[3s - 6] = \frac{2 + 3s^2 + 12}{s^2 + 4} = \frac{3s^2 + 14}{(s^2 + 14)}$

$y(s) = \frac{3s^2 + 14}{(s^2+4)(3s-6)} = \frac{A + B}{(s^2+4)} + \frac{C}{(3s-6)}$

$C: \left.\frac{3s^2 + 14}{s^2 + 4}\right|_{s=2} = \frac{3(2)^2 + 14}{2^2 + 4} = \frac{13}{4}$

$3s^2 + 14 = (A + B)(3s - 6) + C(s^2 + 4)$

$3s^2 + 14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C$

Comparing Co-efficient.

$3 = 3B + C$

$3 = 3B + \frac{13}{4}$

$3A - 6B = 0$

$3A = 6B$

$3A = 6 \times -\frac{1}{12}$

$A = -\frac{1}{6}$

$$y(s) = \frac{-\frac{1}{6} - \frac{1}{12}}{s^2+4} + \frac{\frac{13}{4}}{3s-6}.$$

$$\frac{-\frac{1}{6}}{s^2+4} - \frac{\frac{1}{12}s}{s^2+4} + \frac{\frac{13}{4}}{3s-6}$$

$= -\frac{1}{6} \cdot \frac{1}{s^2+4^2} - \frac{1}{12} \cdot \frac{s}{s^2+2^2} + \frac{13}{4} \cdot \frac{1}{3(s-2)}$

$= -\frac{1}{6} \cdot \frac{1}{2}\left[\frac{2}{s^2+2^2}\right] - \frac{1}{12}\left[\frac{s}{s^2+2^2}\right] + \frac{13}{12}\left[\frac{1}{s-2}\right]$

$y(t) = -\frac{1}{12} \sin 2t - \frac{1}{12} \cos 2t + \frac{13}{12} e^{2t}.$

$y(t) = \frac{1}{12}\left[-\sin 2t - \cos 2t + 13e^{2t}\right]$

$y(t) = \frac{1}{12}\left[15e^{2t} - \cos 2t - \sin 2t\right].$

3) $\frac{dy}{dt} - 4y = 8$ given that $t=0$ $y=2$ $y(0)=2$

$y' - 4y = 8$

$sy(s) - y(0) - 4y(s) = \frac{8}{s}.$

$sy(s) - 2 - 4y(s) = \frac{8}{s}$

$y(s)[s-4] = \frac{8}{s} + \frac{2}{1} = \frac{8+2s}{s}$

$$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}.$$

$$A : \left.\frac{8+2s}{s-4}\right|_{s=0} = \frac{8}{-4} = -2$$

$$B : \left.\frac{8+2s}{s}\right|_{s=4} = \frac{8+2(4)}{4} = 4$$

$y(s) = -\frac{2}{s} + \frac{4}{s-4}$

$y(t) = -2 + 4e^{4t}$

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4) $d^2y/dx^2 - 2dy/dx + 5y = e^{2t}$ at $t=0, y=2, y'=1, y(s)=2, y(0)=1$

$y'' - 2y' + 5y = e^{2t}$

$(s^2 y(s) - sy(s) - y'(0)) - 2(sy(s) - y(s)) + sy(s) = 1/s-2$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = 1/s-2$

$y(s)[s^2 - 2s + 5] = 1/s-2 + 2s/1 - 3/1 = \frac{1 + 2s(s-2) - 3(s-2)}{s-2}$

$y(s)[s^2 - 2s + 5] = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2} = \frac{2s^2 - 7s + 7}{s-2}$

$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$

$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5}$

$A : \left.\frac{2s^2 - 7s + 7}{s^2 - 2s - 5}\right|_{s=-2} = \frac{2(2)^2 - 7(2) + 7}{2^2 - 2(2) + 5} = 1/5$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$

$2s^2 + 5 + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$2 = A + B$

$2 = 1/5 + B$

$B = 9/5$

$7 = 5A - 2C$

$7 = 5(1/5) - 2C$

$7 - 1 = 2C$

$C = -3$

$y(s) = \frac{1/5}{s-2} + \frac{9s/5 - 3}{s^2 - 2s + 5} - 1/5 \cdot \frac{1}{s-2} + \frac{9s/5}{(s+1)^2 + 4} - \frac{3}{(s+1)^2}$

$y(s) = 1/5 \cdot 1/s-2 + 9/5 \frac{s-1+1}{(s+1)^2 + 4} - \frac{3}{(s+1)^2 + 2^2}$

$y(s) = 1/5 \cdot 1/s-2 + 4/5 \frac{(s-1)}{(s+1)^2 + 4} - 3/(s+1)^2 + 2^2$

$y(s) = 1/5 \cdot 1/s-2 + 9/5 \frac{(s+1)}{(s+1)^2 + 2^2} - 1/(s+1)^2 + 4 - 3/(s+1)^2 + 2$

$y(s) = 1/5 \cdot 1/s-2 + 9/5 \frac{s+1}{(s+1)^2 + 2} - \frac{4}{(s+1)^2 + 2^2} \cdot 2/2$

$y(t) = 1/5\, e^{2t} + 9/5\, e^{-t} \cos 2t$

$2e^{-t} \sin 2t$.

$y(t) = 1/5\,[e^{2t} + 9e^{-t}\cos 2t - 10e^{-t}\sin 2t]$.

5. $\frac{d^2y}{dx} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0$ $y=0$ $y'=2$

$y'' - 6y' + 8y = e^{3t}$

$s^2y(s) - sy(s) - y'(0) - 6[sy(s) - y(0)] + 8y(s) = 1/s-3$

$s^2y(s) - 2 - 6sy(s) + 8y(s) = 1/s-3$

$y(s)[s^2 - 6s + 8] = 1/s-3 + 2/1 \Rightarrow \frac{1+2(s-3)}{s-3}$

$y(s)[s^2-6s+8] = \frac{1+2s-6}{s-3}$

$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{2s-5}{(s-3)(s-2)(s-4)} \Rightarrow \frac{A}{(s-3)} + \frac{B}{(s-2)} - \frac{C}{s-4)}$

$A \cdot \frac{2s-5}{2-2)(s-4}\Big|_{s=3} = \frac{2(5)-5}{(3-2)(3-4} = -1$

$B: \frac{2s-5}{(s-2)(s-4}\Big|_{s=2} = \frac{2(2)-5}{(2-3)(2-4)} = -1/2$

$C: \frac{2s-5}{s-3)(s-2}\Big|_{s=4} = \frac{2(4)-5}{(4-3)(4-3)} = 3/2$

$y(s) = \frac{-1}{s-3} - 1/2 = 1/s-2 + 3/2 \cdot \frac{1}{s-4}$

$y(t) = -e^{3t} - 1/2 e^{2t} + 3/2 e^{4t}$