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Eula 301

1)

$$y'/dt + 3y = e^{-2t}$$

$$y'(t) + 3y(t) = e^{-2t}$$

$$L[y'(t)] = 3Y(s) - y(0)$$

$$L[e^{-2t}] = Y(s), L^{-1}[e^{-2t}] = \frac{1}{6t2}$$

$$5Y(s) - y(0) + 3Y(s) = \frac{1}{5t2}$$

$$(5-3)Y(s) - 2 = \frac{1}{5t2}$$

$$(5-3)Y(s) = \frac{1}{5t2} + 2 = \frac{1+2s+4}{5t2} = \frac{2s+5}{5t2}$$

$$Y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$2(s+3) + 5 = A(s+3) + B(s+2) \Rightarrow A = -1/5$$

$$2(s+3) + 5 = B(s+2) \Rightarrow B = 11/5$$

$$Y(s) = \frac{-1}{5(s+2)} + \frac{11}{5(s+3)} = -\frac{1}{5}e^{-2t} + \frac{11}{5}e^{-3t}$$

2) $3y'' - 6y' = \sin 2t = 3y'(t) - 6y(t)$

$$L[3y''(t)] = 3Y(s) - y(0)$$

$$L[3y'(t)] = 3Y(s)$$

$$L[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$

$$3s + Y(s) - 3Y(s) - 6Y(s) = \frac{2}{s^2+4}$$

$$s^2+4$$

$$Y(s)[3s-6] = \frac{2}{s^2+4} + 3 = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$$

$$Y(s) = \frac{3s^2 + 14}{(s-1)(s^2+4)} = \frac{A}{s-1} + \frac{B}{s^2+4}$$

$$3s^2 + 14 = A(s^2+4) + B(s-1)$$

$$4A - 6B = 14$$

$$-16B = 14 - 12$$

$$B = -\frac{1}{3}$$

$$Y(s) = \frac{3}{s-1} - \frac{1}{s^2+4}$$

$$= \frac{3e^{2t}}{s-1} - \frac{1}{s^2+2^2}$$

ii) $sy' - 4y = 8$

$$Y(s) - 4Y(s) = \frac{8}{s}$$

$$Y(s) - 4Y(s) = \frac{8}{s}$$

$$sY(s) - 4Y(s) = \frac{8}{s}$$

$$(s-4)Y(s) = \frac{8}{s} + 2 = \frac{8+2s}{s}$$

$$Y(s) = \frac{2s+8}{s(s-4)}$$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s+8 = A(s-4) + B(s)$$

$$2(0)+8 = A(0-4) \Rightarrow A = -2$$

$$2(4)+8 = A(4-4) + B(4)$$

$$B = 4$$

$$L^{-1} \left[\frac{-2}{s} + \frac{4}{s-4} \right] = -2 + 4e^{4t}$$

$$2) \quad \frac{d^2 y}{dt^2} - 2 \frac{dy}{dt} + 5y = e^{2t}$$

$$y''(t) - 2y'(t) + 5y(t) = e^{2t}$$

$$\mathcal{L}\{y''(t)\} = s^2 Y(s) - sY(s) - Y'(s)$$

$$\mathcal{L}\{y'(t)\} = sY(s) - Y(s)$$

$$\mathcal{L}\{y(t)\} = Y(s)$$

$$s^2 Y(s) - sY(s) - Y'(s) - 2sY(s) + 2Y(s) + 5Y(s) = \frac{1}{s-2}$$

$$\mathcal{L}\{s^2 - 2s + 5\} Y(s) + (2-s) Y'(s) = \frac{1}{s-2}$$

$$(s^2 - 2s + 5) Y(s) = \frac{1}{s-2} - (2-s) 2e^{2t}$$

$$= \frac{1}{s-2} + 2s - 4e^{2t} = \frac{2}{s-2} + 2s - 3$$

$$= \frac{2 + (2s-3)(s-2)}{(s-2)} = \frac{2 + 2s^2 - 4s - 3s + 6}{s-2}$$

$$Y(s) = \frac{2s^2 - 7s + 8}{(s-2)(s^2 - 2s + 5)}$$

$$(s-2)(s^2 - 2s + 5)$$

$$2s^2 - 7s + 8 = \frac{A}{s-2} + \frac{B}{s^2 - 2s + 5}$$

$$2s^2 - 7s + 8 = A(s^2 - 2s + 5) + B(s-2)$$

$$A = 2$$

$$-2A + B = -7$$

$$B = 2A - 7$$

$$B = 2(2) - 7 = 4 - 7 = -3$$

$$= \frac{2}{s-2} - \frac{3}{s^2 - 2s + 5}$$

$$= 2e^{2t} - \frac{3}{5} e^{2t} \sin 2t$$

$$2) \quad \frac{d^2 y}{dx^2} - 6 \frac{dy}{dt} + 8y = e^{3t}$$

$$s^2 Y(s) - 3Y(s) - Y'(s) - 6sY(s) + 6Y(s) + 8Y(s) = \frac{1}{s-3}$$

$$[s^2 - 6s + 8] Y(s) = \frac{1}{s-3} + 2 = \frac{1+2s-6}{s-3} = \frac{2s-5}{s-3}$$

$$Y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$2s-5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-3)(s-2)$$

$$2s-5 = A(s^2-6s+8) + B(s^2-7s+12) + C(s^2-5s+6)$$

$$2(3)-5 = A(3-2)(3-4) \Rightarrow A = -1$$

$$2(4)-5 = C(4-3)(4-2) \Rightarrow C = \frac{3}{2}$$

$$-6A - 7B - 5C = 7$$

$$-6(-1) - 7(B) = 7 + 5\left(\frac{3}{2}\right) = 2$$

$$-7B = 2 + \frac{15}{2} - 6 = \frac{4+15-12}{2} = \frac{7}{2} \Rightarrow B = -\frac{1}{2}$$

$$L^{-1} \left[\frac{1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)} \right]$$

$$= -e^{3t} - \frac{1}{2} e^{2t} + \frac{3}{2} e^{4t}$$