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15/ENG06/054
ENG 381
Mechanical Engineering

① $\frac{dy}{dt} + 3y = e^{-2t}$, given that at $t=0, y=2$

Solution.

$$sY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$$

when $y(0) = 2$, $t = 0$

$$sY(s) + 3Y(s) - 2 = \frac{1}{s+2}$$

$$Y(s)(s+3) = \frac{1}{s+2} + 2 = \frac{1+2(s+2)}{s+2} = \frac{1+2s+4}{s+2}$$

$$Y(s)(s+3) = \frac{5+2s}{s+2}$$

$$Y(s) = \frac{5+2s}{(s+2)(s+3)}$$

$$\frac{5+2s}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$5+2s = A(s+3) + B(s+2)$$

$s = -2$

$5-4 = A$; $A = 1$

$s = -3$

$-1 = -B$; $B = 1$

$$\frac{5+2s}{(s+2)(s+3)} = \frac{1}{(s+2)} + \frac{1}{(s+3)}$$

$$\mathcal{L}^{-1} Y(s) = y(t)$$

$$y(t) = \mathcal{L}^{-1}\frac{1}{s+2} + \mathcal{L}^{-1}\frac{1}{s+3} = e^{-2t} + e^{-3t}$$

⑤ $3\frac{dy}{dt} - 6y = \sin 2t$ given that $t=0$ $y=1$

$3sY(s) - 3y(0) - 6Y(s) = \frac{2}{s^2+4}$

$3sY(s) - 6Y(s) - 3y(0) = \frac{2}{s^2+4}$

$3sY(s) - 6Y(s) - 3(1) = \frac{2}{s^2+4}$

$3Y(s)(3s-6) = \frac{2}{s^2+4} + 3 = \frac{2+3(s^2+4)}{s^2+4} = \frac{2+3s^2+12}{s^2+4}$

$Y(s)(3s-6) = \frac{3s^2+14}{s^2+4}$

$Y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)}$

$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A}{3s-6} + \frac{Bs+C}{(s^2+4)}$

$3s^2+14 = A(s^2+4) + (Bs+C)(3s-6)$

at $s = 2$

$12+14 = A(8)$

$8A = 26$

$A = 13/4$

at $s = 0$

$14 = 4A - 6C = 4\left(\frac{13}{4}\right) - 6C = 13 - 6C$

$6C = 13 - 14 = -1$

$C = -1/6$

at $s = 1$,

$3+14 = 5A + (B+C)(-3) = 5A - 3B - 3C$

$17 = 5\left(\frac{13}{4}\right) - 3(B) - 3\left(-\frac{1}{6}\right) = \frac{65}{4} - 3B + \frac{1}{2}$

$17 - \frac{1}{2} - \frac{65}{4} = -3B$

$B = 1/12$

$$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{13}{4}\left(\frac{1}{3s-6}\right) + \frac{\frac{1}{12}s + -\frac{1}{6}}{s^2+4} = \frac{13}{4(s-6)} + \frac{s-2}{12(s^2+4)}$$

$$\mathcal{L}^{-1}Y(s) = \mathcal{L}^{-1}y(t)$$

$$y(t) = \mathcal{L}^{-1}\frac{13}{4(3s-6)} + \mathcal{L}^{-1}\frac{s-2}{12(s^2+4)} = \mathcal{L}^{-1}\frac{13}{12(s-2)} + \mathcal{L}^{-1}\frac{s-2}{12(s^2+4)}$$

$$= \frac{13}{12}e^{2t} - \mathcal{L}^{-1}\frac{s}{12(s^2+4)} - \mathcal{L}^{-1}\frac{2}{12(s^2+4)}$$

$$= \frac{13}{12}e^{2t} + \frac{1}{12}\cos 2t \quad \Phi - \frac{1}{12}\sin 2t = \frac{13}{12}e^{2t} + \frac{1}{12}(\cos 2t -$$

(3) $\frac{dy}{dt} - 4y = 8$ given that at $t=0$ $y=2$

solution.

$$sY(s) - y(0) - 4Y(s) = 8/s$$

$$sY(s) - 4Y(s) - y(0) = 8/s$$

$$y = 2 \quad t = 0$$

$$Y(s)(s-4) - 2 = 8/s$$

$$Y(s)(s-4) = 8/s + 2 = \frac{8+2s}{s}$$

$$Y(s) = \frac{8+2s}{s(s-4)}$$

$$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$8+2s = A(s-4) + Bs$$

at $s=0$

$$8 = -4A$$

$$A = -2.$$

at $s=4$

$$16 = 4B$$

$$B = 4$$

$$\frac{8+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{s-4}$$

$$\mathcal{L}^{-1}Y(s) = y(t)$$

$$y(t) = \mathcal{L}^{-1}\frac{4}{s-4} - \mathcal{L}^{-1}\frac{2}{s}$$

$$y = 4e^{4t} - 2.$$

(4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ at $t=0, y=2, y'=1$

solution.

$$s^2Y(s) - sy(0) - y'(0) - 2sY(s) + 2y(0) + 5Y(s) = \frac{1}{s-2}$$

$$s^2Y(s) - 2sY(s) + 5Y(s) - sy(0) - y'(0) + 2y(0) = \frac{1}{s-2}$$

$$Y(s)(s^2-2s+5) - sy(0) - y'(0) + 2y(0) = \frac{1}{s-2}$$

at $y=2$ & $y'=1$

$$Y(s)(s^2-2s+5) - 2s - 1 + 4 = \frac{1}{s-2}$$

$$Y(s)(s^2-2s+5) - 2s + 3 = \frac{1}{s-2}$$

$$Y(s)(s^2-2s+5) = \frac{1}{s-2} + 2s - 3 = \frac{1+2s(s-2)-3(s-2)}{s-2}$$

$$Y(s)(s^2+2s+5) = \frac{1+2s^2-4s-3s+6}{s-2} = \frac{2s^2+7-7s}{s-2}$$

$$y(s) = \frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{2s^2-7s+7}{(s-2)}$$

⑤ $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ $t=0, y=0, y'=2$

$s^2y(s) - sy(0) - y'(0) - 6sy(s) - 6y(0) + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - 6sy(s) + 8y(s) - sy(0) - y'(0) - 6y(0) = \frac{1}{s-3}$

$y(s)(s^2-6s+8) - 2 = \frac{1}{s-3}$

$y(s)(s^2-6s+8) = \frac{1}{s-3} + 2 = \frac{1+2(s-3)}{s-3} = \frac{1+2s-6}{s-3}$

$y(s)(s^2-6s+8) = \frac{2s-5}{s-3}$

$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{2s-5}{(s-3)(s-4)(s-2)}$

$\frac{2s-5}{(s-3)(s-4)(s-2)} = \frac{A}{(s-3)} + \frac{B}{(s-4)} + \frac{C}{(s-2)}$

$2s-5 = A(s-4)(s-2) + B(s-3)(s-2) + C(s-3)(s-4)$

let $s=4$

$8-5 = 2B$

$B = 3/2$

let $s=2$

$4-5 = 2C$

$C = -1/2$

let $s=3$

$6-5 = -A$

$A = -1$

$$\frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{-1}{s-3} + \frac{3}{2(s-4)} + \frac{-1}{2(s-2)}$$

$$\mathcal{L}^{-1}\,Y(s) = y(t)$$

$$y(t) = \mathcal{L}^{-1}\frac{-1}{s-3} + \mathcal{L}^{-1}\frac{3}{2(s-4)} + \mathcal{L}^{-1}\frac{-1}{2(s-2)}$$

$$y(t) = -e^{3t} + \frac{3}{2}e^{4t} - \frac{1}{2}e^{2t}$$