EZIMUOBOR GREAT
MECHATRONICS ENGR
15/SCI03/006
ENG 381

i) $\frac{dy}{dt} + 8y = e^{-2t}$

$y(t) + 8y(t) = e^{-2t}$

$L[y'(t)] = sy(s) - y(s)$

$L[y(t)] = y(s)$; $L^{-1}[e^{-2t}] = 1/s+2$

$3y(s) - y(s) + 5y(s) = 1/s+2$

$(s-3)\,y(s) - 2 = 1/s+2$

$(s-3)Y(s) = 1/s+2 + 2 = \frac{1+2s+4}{s+2} = \frac{2s+5}{s+2}$

~~$Y(s) = \frac{2s+5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$~~

~~$2(-2)+5 = A(-2-3)$~~

$Y(s) = \frac{2s+5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$

$2(-2)+5 = A(-2-3) \Rightarrow A = 1/-5$

$2(3)+5 = B(3+2) \Rightarrow B = 11/5$

$y(s) = \frac{-1}{s(s+2)} + \frac{11}{s(s-3)} = \frac{-1}{s}e^{-2t} + \frac{11}{s}e^{3t}$

ii) $3dy/dt - 6y = 5\sin 2t = 3y'(t) - 6y(t)$

$L[y'(t)] = sy(s) - y(0)$

$L[y(t)] = y(s)$

$L(\sin 2t) = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$

$3s + Y(s) - 3y(0) - 6y(0) = 2/s^2+4$

$Ys(3s-6) = \frac{2}{s^2+4} + 5 = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$

$Y(s) = \frac{3s^2+14}{(3s+6)(s^2+4)} + A/3s-6 + B/s^2+4$

$$Y(s) = \frac{3s^2+14}{(3s-6)(s^2+4)} = \frac{A}{3s-6} + \frac{B}{s^2+4}$$

$$3s^2+14 = A(s^2+4) + B(3s-6)$$

$$4A - 6B = 14$$

$$-6B = 14 - 12$$

$$B = -1/3$$

$$Y(s) = \frac{B}{[B](s-2)} - \frac{1}{3(s^2+4)}$$

$$= e^{2t} - \frac{1}{6}\sin 2t$$

iii) $dy/dt - dy = 8$

$$y'(t) - 4y(t) = 8$$

$$y'(t) - 4y(t) = 8$$

$$sy(s) - y(0) - 4y(s) = 8/s$$

$$(s-4)(y(s)) = 8/s + 2 = \frac{8+2s}{s}$$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s+8 = A(s-4) + B(s)$$

$$2(0)+8 = A(0-4) \Rightarrow A = -2$$

$$2(4)+8 = A(4-4) + B(4)$$

$$B = 4$$

$$L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right] = -2 + 4e^{4t}$$

iv) $\frac{d^2y}{dt^2} - \frac{2dy}{dt} + 5y = e^{2t}$

$$y''(t) - 2y'(t) + 5y(t) = e^{2t}$$

$$L[y''(t)] = s^2y(s) - sy(0) - y'(0)$$

$$L[y'(t)] = sy(s) - y(s)$$

$$L[y(t)] = y(s)$$

$$s^2y(s) - sy(s) - y'(0) - 2sy(s) + 2y(s) + 5y(s) = 4/s^2$$

$(s^2-2s+5)Y(s)+(2-s)y_0-y'(0)=1/s-2$

$(s^2-2s+5)Y(s)=1/s-2-[2-s]2+1$

$=\frac{1}{s-2}+2s-4+1=\frac{1}{s-2}+2s-3$

$=\frac{1+(2s-3)(3-2)}{(s-2)}=\frac{1+2s^2+4s-3s+6}{s-2}$

$Y(s)=\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)}$

$2s^2-7s+7=\frac{A}{s-2}+\frac{B}{s^2-2s+5}$

$2s^2-7s+7=A(s^2-2s+5)+B(s-2)$

$A=2$

$-2A+B=-7$

$B=-7+4=-3$

$=\frac{2}{s-2}-\frac{3}{s-2s+5}$

$=2e^{2t}-7/8+\sin 2t$

v) $d^2y/dt^2-6\frac{dy}{dt}+8y=e^{3t}$

$s^2Y(s)-sy(0)+y'(0)-6sY(s)+6Y(0)+8Y(s)=1/s-3$

$[s^2-6s+8)Y(s)+(6-s)y(0)-y'(0)=1/s-3$

$[s^2-6s+8]\cdot Y(s)=\frac{1}{s-3}+2=\frac{1+2s-6}{s-3}=\frac{2s-5}{s-3}$

$Y(s)=\frac{2s-5}{(s-3)(s-2)(s-4)}=\frac{A}{s-3}+\frac{B}{s-2}+\frac{C}{s-4}$

$2s-5=A(s-2)(s-4)+B(s-3)(s-4)+C[(3-3)(3-2)]$

$2s-5=A[s^2-6s+8]+B(s^2-7s+12)+C[s^2-6s+6]$

$2[3]-5=A[3-2][3-4]\Rightarrow A=-1$

$2[4]-5=C[4-3](4-2]\Rightarrow C=3/2$

$-6A-7B-5C=7$

$-6[-1]-7B-5[3/2]=2$

$$-7B = 2 + \frac{15}{2} - 6 = \frac{4 + 15 - 12}{2} = \frac{7}{12}$$

$$L^{-1}\left[\frac{1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)}\right]$$

$$= -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$