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15IENG08/03+

MECHANICAL ENGINEERING

ENG 381

1.)  $\frac{dy}{dt} + 3y = e^{-2t}$ , given that at  $t=0, y=2$

$$L\left\{\frac{dy}{dt}\right\} = sy(s) - y(0)$$

$$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$$

$$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$$

$$sy(s) + 3y(s) - 2 = \frac{1}{s+2}$$

$$y(s) [s+3] = \frac{1}{s+2} + 2$$

$$y(s) [s+3] = \frac{1+2(s+2)}{(s+2)}$$

$$y(s) = \frac{1+2s+4}{(s+2)(s+3)} = \frac{2s+5}{(s+2)(s+3)}$$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$2s+5 = A(s+3) + B(s+2)$$

When  $s+3=0$

$$s = -3$$

$$2(-3)+5 = A(-3+3) + B(-3+2)$$

$$-6+5 = A(0) + B(-1)$$

$$-1 = -B$$

$$B = 1$$

When  $s+2=0$

$$s = -2$$

$$2(-2)+5 = A(-2+3) + B(-2+2)$$

$$-4+5 = A(1) + B(0)$$

$$1 = A$$

$$\therefore A = 1$$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{1}{s+2}\right\} + L^{-1}\left\{\frac{1}{s+3}\right\}$$

$$= e^{-2t} + e^{-3t}$$

2.  $3 \frac{dy}{dt} - 6y = \sin 2t$ , given that at  $t=0$ ,  $y=1$

$$3L\left\{\frac{dy}{dt}\right\} = 3\{sy(s) - y(0)\}$$

$$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+4}$$

$$3sy(s) - 3 - 6y(s) = \frac{2}{s^2+4}$$

$$3sy(s) - 6y(s) - 3 = \frac{2}{s^2+4}$$

$$y(s)(3s-6) = \frac{2}{s^2+4} + 3$$

$$y(s)(3s-6) = \frac{2+3(s^2+4)}{s^2+4} = \frac{2+3s^2+4}{(s+2)^2}$$

$$y(s) = \frac{2+3s^2+12}{(s+2)^2(3s-6)} = \frac{3s^2+14}{(s+2)^2(3s-6)}$$

$$\frac{3s^2+14}{(s+2)^2(3s-6)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{3s-6}$$

$$3s^2+14 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$$

when  $s+2=0$   $s=-2$

$$3(-2)^2+14 = A(0) + B[3(-2)-6] + C(0)$$

$$12+14 = B(-12)$$

$$26 = -12B$$

$$B = -$$

$$2+3(s+2)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$$

$$2+3s^2+12s+12 = As^2 - A2 + 3Bs - 6B + Cs^2 + 4Cs + 4C$$

$$3A+C=3$$

$$3B+4C=12$$

$$-12A-6B+4C=14$$

~~A~~

$$3A = 3-C$$

$$A = \frac{3-C}{3}$$

$$3B+4C=12$$

$$-12\left(\frac{3-C}{3}\right) - 6B + 4C = 14$$

$$-12 + 4C - 6B + 4C = 14$$

$$-6B + 8C = 26$$

$$-18B - 24C = -72$$

$$-18B - 24C = 78$$

$$-48C = -150$$

$$C = \frac{25}{8}$$

$$3B = 12 - 4\left(\frac{25}{8}\right)$$

$$B = -\frac{1}{6}$$

$$3A = 3 - C$$

$$A = \frac{3 - \left(\frac{25}{8}\right)}{3}$$

$$A = -\frac{1}{24}$$

$$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{-\frac{1}{24}}{s-2} - \frac{\frac{1}{6}}{(s+2)^2} + \frac{\frac{25}{8}}{3s-6}$$

$$\mathcal{L}^{-1}\{y(s)\} = \mathcal{L}^{-1}\left\{\frac{-\frac{1}{24}}{(s+2)} - \frac{\frac{1}{6}}{(s+2)^2} + \frac{\frac{25}{8}}{(3s-6)}\right\}$$

$$y = \frac{-1}{24}e^{-2t} - \frac{1}{6}te^{-4t} + \frac{25}{24}e^{3t}$$

$$y = \frac{-1}{6} \left\{ \frac{1}{4}e^{-2t} - te^{-4t} + \frac{25}{4}e^{3t} \right\}$$

3.  $\frac{dy}{dt} - 4y = 8$ , given that at  $t=0$ ,  $y=2$

$$Sy(s) - y(0) - 4y(s) = \frac{8}{s}$$

$$Sy(s) - 4y(s) - 2 = \frac{8}{s}$$

$$y(s)(s-4) = \frac{8+2s}{s}$$

$$\frac{8+2s}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)}$$

$$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

when  $s-4=0 \therefore s=4$

$$8+2s = A(s-4) + Bs$$

$$8+2(4) = A(4-4) + B(4)$$

$$16 = A(0) + 4B$$

$$B = 4$$

$$s=0$$

$$8+2(0) = A(0-4) + B(0)$$

$$8 = A(-4)$$

$$A = -2$$

$$\frac{8+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{s-4}$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{-2}{s}\right\} + 4L^{-1}\left\{\frac{1}{s-4}\right\}$$

$$= -2 + 4e^{4t}$$

4.  $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ , given that at  $t=0$ ,  $y=2$ ,  $y'=1$

$$s^2y(s) - sy(0) - y'(0) - 2sy(s) + 2y(0) + 5y(s) = \frac{1}{s-2}$$

$$s^2y(s) - 2sy(s) + 5y(s) - 2s - 1 + 4 = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 3$$

$$\frac{1 + 2s(s-2) - 3(s-2)}{s-2}$$

$$= \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$$

$$y(s)(s^2 - 2s + 5) = \frac{2s^2 - 7s + 7}{s-2}$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + Bs + C(s-2)$$

$$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

$$s^2: A + B = 2 \quad \text{--- (1)}$$

$$s: -2A - 2B + C = -7 \quad \text{--- (2)}$$

$$: 5A - 2C = 7 \quad \text{--- (3)}$$

From (1)

$$B = 2 - A$$

From (2)

$$-2A - 2(2 - A) + C = -7$$

$$-2A - 4 + 2A + C = -7$$

$$-4 + C = -7$$

$$C = -7 + 4 \quad \therefore C = -3$$

From 3

$$5A - 2(-3) = 7$$

$$5A + 6 = 7$$

$$5A = 7 - 6$$

$$A = \frac{1}{5}$$

Recall

$$B = 2 - A$$

$$B = 2 - \frac{1}{5}$$

$$B = \frac{9}{5}$$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{1/5}{(s-2)} + \frac{9/5s - 3}{(s^2 - 2s + 5)}$$

$$= \frac{1/5}{s-2} + \frac{9/5s}{s^2 - 2s + 5} - \frac{3}{s^2 - 2s + 5}$$

$$= \frac{1/5}{s-2} + \frac{9}{5} \left[ \frac{s-1+1}{(s-1)^2 + 4} \right] - \frac{3}{2} \left[ \frac{2}{(s-1)^2 + 4} \right]$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{1/5}{s-2}\right\} + \frac{9}{5} L^{-1}\left\{\frac{s-1}{(s-1)^2 + 2^2} - \frac{1 \times \frac{3}{2}}{(s-1)^2 + 2^2}\right\}$$

$$- \frac{3}{2} L^{-1}\left\{\frac{2}{(s-1)^2 + 2^2}\right\}$$

$$L^{-1}\left\{\frac{1/5}{s-2} + \frac{9}{5} \left[ \frac{s-1}{(s-1)^2 + 2^2} + \frac{1}{2} \left( \frac{2}{(s-1)^2 + 2^2} \right) \right] - \frac{3}{2} \left[ \frac{2}{(s-1)^2 + 2^2} \right] \right\}$$

$$y = \frac{1}{5} e^{2t} + \frac{9}{5} \left[ e^t \cos 2t + \frac{1}{2} e^t \sin 2t \right] - \frac{3}{2} \left[ e^t \sin 2t \right]$$

5.  $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$  given at  $t=0, y=0, y'=2$

$$s^2 y(s) - s y(0) - y'(0) - 6s y(s) + 6y(0) + 8y(s) = \frac{1}{s-3}$$

$$s^2 y(s) - 6s y(s) + 8y(s) - 2 = \frac{1}{s-3}$$

$$y(s)(s^2 - 6s + 8) = \frac{1}{s-3} + 2$$

$$y(s)(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{s-3} + \frac{Bs+C}{s^2-6s+8}$$

$$2s-5 = A(s^2-6s+8) + Bs+C(s-3)$$

$$2s-5 = As^2 - 6As + 8A + Bs^2 - 3Bs + Cs - 3C$$

$$s^2: A+B=0 \quad \text{--- (1)}$$

$$S: -6A - 3B + C = 2 \quad \text{--- (2)}$$

$$8A - 3C = -3 \quad \text{--- (3)}$$

$$B = -A \quad \text{from (1)}$$

$$-6A + 3A + C = 2$$

$$-3A + C = 2$$

$$8A - 3C = 5$$

$$9A - 3C = -6$$

$$8A - 3C = -5$$

$$A = -1$$

$$B = 1$$

From

$$C = 2 - 3A \quad \therefore C = -1$$

$$\frac{2S - 5}{(S - 3)(S^2 - 6S + 8)} = \frac{-1}{S - 3} + \frac{S - 1}{S^2 - 6S + 8}$$

$$\frac{2S - 5}{(S - 3)(S^2 - 6S + 8)} = \frac{-1}{S - 3} + \frac{S - 1}{(S - 2)(S - 4)}$$

$$\frac{S - 1}{(S - 2)(S - 4)} = \frac{A}{S - 2} + \frac{B}{S - 4}$$

$$S - 1 = A(S - 4) + B(S - 2)$$

$$S - 1 = AS - 4A + BS - 2B$$

$$A + B = 1 \quad \times -4$$

$$-4A - 2B = -1 \quad \times 1$$

$$-4A - 4B = -4$$

$$-4A - 2B = -1$$

$$-2B = -3$$

$$B = \frac{3}{2}$$

$$A = -\frac{1}{2}$$

$$\frac{S - 1}{(S - 2)(S - 4)} = \frac{-\frac{1}{2}}{S - 2} + \frac{\frac{3}{2}}{S - 4}$$

$$6. \frac{2s-5}{(s-2)(s-4)} = \frac{-1}{s-3} + \left( \frac{-\frac{1}{2}}{s-2} + \frac{\frac{3}{2}}{s-4} \right)$$

$$\mathcal{L}^{-1}\{y(s)\} = \mathcal{L}^{-1}\left\{ \frac{-1}{s-3} - \frac{\frac{1}{2}}{s-2} + \frac{\frac{3}{2}}{s-4} \right\}$$

$$y = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$

$$y = -\frac{1}{2} \left[ 2e^{3t} + e^{2t} - 3e^{4t} \right]$$