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15/ES/YG03/009
Civil

1) I.P $y = e^{x^2+x}$

Show that $y'' = y'(2x+1) + 2y$

$y = e^{x^2+x}$

$dy/dx = y'$

$y' = (2x+1)e^{x^2+x}$

$d^2y/dx^2 = y''$

Since $y' = (2x+1)e^{x^2+x}$

$d^2y/dx^2 = y'' = V\frac{du}{dx} + U\,dv/dx$

$= 2(e^{x^2+x}) + 2x+1\,(2x+1e^{x^2+x})$

Since $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$\cancel{\frac{d^2y}{dx^2} = y'' = V^{du}}$

$= y'' = 2(y) + (2x+1)(y')$

$y'' = y'(2x+1) + 2y$

6) Hence prove that

$y^{(n+2)} = (2x+1)y^{n+1} + 2(n+1)y^n$

$y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

Using leibnit theorem

let $w = y''$

$u = y''$

$u^n = y^{n+2}$

$w^n = y^{n+2}$

let $w = y'(2x+1)$

$u = y'$

$u^n = y^{n+1}$ $\quad V = -(2x+1)$

$V' = -2$

$V'' = 0$

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$w^n = u^n v^0 + nu^{n-1} v' + \frac{n(n-1)}{1 \cdot 2} u^{n-2} v''$

$= u^n v^0 + nu^{n-1} v'$

$= -y^{n+1}(2x+1) - 2ny^n$

$w^n = -y^{n+1}(2x+1) - 2ny^n$

Let $w = -2y$

$u = y$ $\quad v = -2$

$u^n = y^n$ $\quad v = 0$

$w^n = u^n v + nu^{n-1} v'$

$w^n = -2y^n$

$\therefore y^{n+2} - (2x+1)y^{n+1} - 2ny^n - 2y^n = 0$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{(n+1)} + (2n+2)y^n$

$y^{n+2} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2) Using the leibnt theorem give that

$y = x^3 e^{4x}$

detamine y^5

$v = x^3$ $\quad u = e^{4x}$

$v' = 3x^2$ $\quad u' = 4e^{4x}$

$v^{(2)} = 6x$ $\quad u^{(2)} = 16e^{4x}$

$v^{(3)} = 6$ $\quad u^{(3)} = 64e^{4x}$

$v^{(4)} = 0$ $\quad u^{(4)} = 256e^{4x}$

$v^{(5)} = 0$ $\quad u^{(5)} = 1024e^{4x}$

$y^5 = u^{(5)} v^{(0)} + nu^{(4)} v^{(1)} + \frac{n(n-1)}{2!} u^{(3)} v^{(2)} + \frac{n(n-1)(n-2)}{3!} u^{(2)} v^{(3)} +$

$\frac{n(n-1)(n-2)(n-3)}{4!} u^{(1)} v^{(4)} + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} u^{(0)} v^{(5)}$

$y^{(5)} = [1024 e^{4x}(x^3)] + [5(256e^{4x})3x^2] + [5 \times 2(64e^{4x})6x] +$

$\left[\frac{5 \times 4 \times 3}{6} \times 16e^{4x} \times 6\right] + [0] + [0]$

$y^5 = 1024e^{4x}x^3 + 1280e^{4x}(3x^2) + 640e^{4x}(6x) + 160e^{4x}(6)$

$y^5 = 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x + 960e^{4x}$

$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

i) $x^2 \, d^2y/dx^2 + x \, dy/dx + y = 0$

$x^2y'' + xy' + y = 0$

let $w = x^2 y''$

$u = y''$ $\qquad V = x^2$

$u^n = y^{n+2}$ $\qquad V' = 2x$

$V'' = 2$

$V''' = 0$

$$w^n = u^n v^0 + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v'' + \frac{n(n-1)(n-2)}{3!}u^{n-3}v'''$$

$$= y^{n+2}x^2 + ny^{n+1}(2x) + \frac{n(n-1)}{2}y^n(2) - 0$$

$w^n = y^{n+2}x^2 + 2xny^{n+1} + n(n-1)y^n$

let $w = xy'$

$u = y'$

$u^n = y^{n+1}$ $\qquad V = x$

$V' = 1$

$V'' = 0$

$$w^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v''$$

$= y^{n+1}x + ny^n(1) + 0$

$w^n = y^{n+1}x + ny^n$

let $w = y$

$w^n = y^n$

$0 = y^{n+2}x^2 + ny^{n+1}2x + (n(n-1))y^n + xy^{n+1} + ny^n + y^n$

$0 = x^2(y^{n+2}) + 2xn(y^{n+1}) + (n^2-n)y^n + x(y^{n+1}) + ny^n + y^n$

$0 = (n^2 - n + n + 1)y^n + (2xn + x)y^{n+1} + x^2(y^{n+2})$

$0 = x^2y^{n+2} + (2xn + x)y^{n+1} + (n^2+1)y^n$

$x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$