Assignment 5

1. $\frac{dy}{dt} + 3y = e^{-2t}$

at $t = 0$ $y = 2$.

$y(0) = 2.$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$y(s)(s+3) - 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2.$

$y(s) = \frac{1 + 2s + 4}{(s+2)(s+3)}$

$y(t) = L^{-1}[y(s)] = L^{-1}\left[\frac{1+2s+4}{(s+2)(s+3)}\right]$

$\frac{1+2s+4}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$s = -2 \Rightarrow 1 + 2(-2) + 4 = A(-2+3)$

$1 = A, \quad A = 1$

$s = -3 \Rightarrow 1 + 2(-3) + 4 = B(-3+2)$

$-1 = -B, \quad B = 1$

$y(t) = L^{-1}\left[\frac{1}{s+2} + \frac{1}{s+3}\right]$

$y(t) = e^{-2t} + e^{-3t}$

ii) $3\dfrac{dy}{dt} - 6y = \sin 2t$

at $t = 0$, $y = 1$

$$3(sy(s) - y(0)) - 6y(s) = \frac{2}{s^2+4}$$

$$3sy(s) - 3(1) - 6y(s) = \frac{2}{s^2+4}$$

$$y(s)(3s-6) = \frac{2}{s^2+4} + 3$$

$$y(s) = \frac{2 + 3(s^2+4)}{(s^2+4)(3s-6)}$$

$$y(s) = \frac{2 + 3s^2 + 12}{(s^2+4)(3s-6)}$$

$$y(t) = L^{-1}\left[\frac{3s^2+14}{(s^2+4)(3s-6)}\right]$$

$$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{As+B}{(s^2+4)} + \frac{C}{(3s-6)}$$

$$3s^2+14 = As + B(3s-6) + C(s^2+4)$$

$$3s^2+14 = 3As^2 - 6As + 3Bs - 6B + Cs^2 + 4C$$

$$3s^2+14 = s^3(3A+C) + s(-6A+3B) - 6B + 4C$$

$3A + C = 3 \cdots (1)$, $\quad C = 3 - 3A$

$3B - 6A = 0 \cdots (2)$ $\quad 4(3-3A) - 6B = 14$

$4C - 6B = 14 \cdots (3)$ $\quad 12 - 12A - 6B = 14$

$A = -\dfrac{1}{12}$, $B = -\dfrac{1}{6}$, $C = \dfrac{13}{4}$

$$\frac{3s^2+14}{(s^2+4)(3s+6)} = -\frac{s}{12(s^2+4)} + \frac{1}{6(s^2+4)} + \frac{13}{4(3s-6)}$$

$$y(t) = -\frac{1}{12}\cos 2t - \frac{1}{6}\sin 2t + \frac{13}{4}e^{+2t}$$

iii) $\frac{dy}{dt} - 4y = 8$

at $t=0$, $y=2$

$y(0) = 2.$

$$sy(s) - y(0) - 4y(s) = \frac{8}{s}$$

$$y(s)(s-4) - 2 = \frac{8}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)}$$

$$y(t) = L^{-1}[y(s)] = L^{-1}\left[\frac{8+2s}{s(s-4)}\right]$$

$$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$s=0 \Rightarrow \quad 8+2(0) = A(0-4)$

$8 = -4A, \quad A = -2.$

$s=4 \Rightarrow \quad 8+2(4) = B(4)$

$16 = 4B, \quad B = 4$

$$y(t) = L^{-1}\left[-\frac{2}{s} + \frac{4}{s-4}\right]$$

$$y(t) = -2 + 4e^{4t}$$

iv) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

at $t = 0,\ y = 2,\ y' = 1$

$y(0) = 2,\quad y'(0) = 1$

$s^2y(s) - sy(0) - y'(0) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) - 2s - 1 + 2 = \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{1}{s-2} - 1 + 2s$

$y(s) = \frac{1 - (s-2) + 2s(s-2)}{(s-2)(s^2-2s+5)}$

$y(s) = \frac{1 - s + 2 + 2s^2 - 4s}{(s-2)(s^2-2s+5)} = \frac{2s^2 - 5s + 3}{(s-2)(s^2-2s+5)}$

$y(t) = L^{-1}[y(s)] = L^{-1}\left[\frac{2s^3 - 5s + 3}{(s-2)(s^2-2s+5)}\right]$

$\frac{2s^2 - 5s + 3}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs + C}{s^2-2s+5}$

$2s^2 - 5s + 3 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$2s^2 - 5s + 3 = s^2(A + B) + s(-2A - 2B + C) + 5A - 2C$

$A + B = 2 \quad \ldots\ (1)$

$-2A - 2B + C = -5 \quad \ldots\ (2)$

$5A - 2C = 3 \quad \ldots\ (3)$

$A = \frac{1}{5},\quad B = \frac{9}{5},\quad C = -1$

v) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$.

$t=0 \quad y=0 \quad y'=2$

$y(0)=0, \quad y'(0)=2$

$s^2y(s) - sy(0) - y'(0) - 6(sy(s) - y(0)) + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - s(0) - 2 - 6sy(s) + 6(0) + 8y(s) = \frac{1}{s-3}$

$y(s)(s^2 - 6s + 8) - 2 = \frac{1}{s-3}$

$y(s) = \frac{1 + 2(s-3)}{(s-3)(s^2-6s+8)} = \frac{1+2s-6}{(s-3)(s-4)(s-2)}$

$y(t) = L^{-1}[y(s)] = L^{-1}\left[\frac{2s-5}{(s-2)(s-3)(s-4)}\right]$

$\frac{2s-5}{(s-2)(s-3)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-3)} + \frac{C}{(s-4)}$

$s=2 \Rightarrow \quad 2(2)-5 = A(2-3)(2-4)$

$-1 = 2A, \quad A = -1/2$

$s=3 \Rightarrow \quad 2(3)-5 = B(3-2)(3-4)$

$1 = -B, \quad B = -1$

$s=4 \Rightarrow \quad 2(4)-5 = C(4-2)(4-3)$

$3 = 2C, \quad C = \frac{3}{2}$

$y(t) = L^{-1}\left[-\frac{1}{2}\cdot\frac{1}{(s-2)} - \frac{1}{(s-3)} + \frac{3}{2}\cdot\frac{1}{(s-4)}\right]$

$y(t) = -\frac{1}{2}e^{2t} - e^{3t} + \frac{3}{2}e^{4t}$

$y(t) = \frac{1}{2}\left(3e^{4t} - 2e^{3t} - e^{2t}\right)$