

15/ENCR07/010

BATTA NIKAL-ABMSI 4

PETROLEUM ENG

$$(1) \frac{dy}{dt} + 3y = e^{-2t} \text{ at } t=0, y=2$$

$$sY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$$

$$Y(s) = \frac{1}{(s+2)(s+3)} = \frac{2}{s+3}$$

$$Y(s) = \frac{1 \cdot s + 4}{(s+2)(s+3)}$$

$$Y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$2s+5 = A(s+3) + B(s+2)$$

$$2s+5 = (A+B)s + 3A+2B$$

$$A+B = 2$$

$$3A+2B = 5$$

$$A = 2-B$$

$$3(2-B) + 2B = 5$$

$$6 - 3B + 2B = 5$$

$$6 - B = 5$$

$$B = 1$$

$$A = 2 - B$$

$$A = 2 - 1$$

$$A = 1$$

$$\therefore Y(s) = \frac{1}{s+2} + \frac{1}{s+3}$$

$$y(t) = e^{-2t} + e^{-3t}$$

$$(2) 3 \frac{dy}{dt} - 6y = 5 \sin 2t$$

$$\frac{dy}{dt} - 2y = \frac{5 \sin 2t}{3}$$

$$sY(s) - y(0) - 2Y(s) = \frac{2}{3s^2+12}$$

$$y(s)(s-2) - 1 = \frac{2}{3s^2+12}$$

$$y(s)(s-2) = \frac{2}{3s^2+12} + 1$$

$$y(s) = \frac{2}{3s^2+12} + \frac{1}{s-2}$$

$$y(s) = \frac{2s-4 + 3s^2+12}{(3s^2+12)(s-2)}$$

$$y(s) = \frac{3s^2+2s+8}{(3s^2+12)(s-2)} = \frac{As+B}{3s^2+12} + \frac{C}{s-2}$$

$$3s^2+2s+8 = As^2-2As+Bs-2B + 3Cs^2+12C$$

$$A+3C = 3 \quad \text{--- } \textcircled{A} \times 4$$

$$B-2A = 2$$

$$12C-2B = 8$$

$$B = 2+2A$$

$$12C - 2(2+2A) = 8$$

$$12C - 4 - 4A = 8$$

$$-4A + 12C = 12$$

$$4A + 12C = 12$$

$$24C = 24$$

$$C = 1$$

$$A+3 = 3$$

$$A = 0$$

$$B = 2+0$$

$$B = 2$$

$$\therefore y(s) = \frac{2}{3s^2+12} + \frac{1}{s-2}$$

$$y(s) = \frac{1}{3} \left( \frac{2}{s^2+2^2} \right) + \frac{1}{s-2}$$

$$y(t) = \frac{1}{3} \sin 2t + e^{2t}$$

$$(iii) \frac{dy}{ds} - 4y = 8$$

$$s y(s) - y(0) - 4y(s) = \frac{8}{s}$$

$$y(s)(s-4) - 2 = \frac{8}{s}$$

$$y(s) = \frac{8}{s(s-4)} + \frac{2}{(s-4)}$$

$$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$8+2s = As + Bs - 4A$$

$$A+B = 2$$

$$-4A = 8$$

$$A = -2$$

$$-2+B = 2$$

$$B = 4$$

$$y(s) = \frac{-2}{s} + \frac{4}{s-4}$$

$$y(s) \Rightarrow y(t) = -2 + 4e^{4t}$$

$$(iv) s^2 y(s) - s y(0) - y'(0) - 2s y(s) + 2y(0) + 5y(s) = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) - 2s + 4 - 1 = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) - 2s + 3 = \frac{1}{s-2}$$

$$y(s) = \frac{1}{(s-2)(s^2-2s+5)} + \frac{2s-3}{s^2-2s+5}$$

$$y(s) = \frac{1 + 2s^2 - 4s - 3s + 6}{(s-2)(s^2-2s+5)}$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs+C}{s^2-2s+5}$$

$$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

$$2s^2 - 7s + 7 = (A+B)s^2 + (-2A-2B+C)s + 5A-2C$$

$$A+B=2$$

$$-2A-2B+C=-7$$

$$5A-2C=7$$

$$\cancel{A=2-B} \quad B=2-A$$

$$\rightarrow -2A-4+2A+C=-7$$

$$C=-3$$

$$5A+6=7$$

$$5A=1$$

$$A=\frac{1}{5}$$

$$B=2-\frac{1}{5}$$

$$B=\frac{9}{5}$$

$$\therefore y(s) = \frac{1}{5(s-2)} + \frac{\frac{9}{5}s-3}{s^2-2s+5}$$

$$y(s) = \frac{1}{5} \left( \frac{1}{s-2} + \frac{9s-15}{s^2-s-5+1+4} \right)$$

$$y(s) = \frac{3}{5} \left( \frac{3}{s-2} + \frac{3s-5}{(s-1)^2+2^2} \right)$$

$$y(s) = \frac{3}{5} \left( \frac{3}{s-2} + \frac{3s-3-2}{(s-1)^2+2^2} \right)$$

$$y(s) = \frac{3}{5} \left( \frac{3}{s-2} + \frac{3(s-1)}{(s-1)^2+2^2} - \frac{2}{(s-1)^2+2^2} \right)$$

$$y(t) = \frac{3}{5} \left( 3e^{+2t} + \frac{3}{5} \cos 3e^t \cos 2t - e^t \sin 2t \right)$$

$$(v) \quad s^2 y(s) - sy(0) - y'(0) - 6y(s) + 6y(0) + 8y(s) = \frac{1}{s-3}$$

$$y(s)(s^2 - 6s + 8) - 2 = \frac{1}{s-3}$$

$$y(s) = \frac{1}{s-3(s^2-6s+8)} + \frac{2}{s^2-6s+8}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-4s-2s+8)}$$

$$y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$2s-5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-3)(s-2)$$

~~$$2s-5$$~~ let  $s=2$

$$-1 = B(-1)(-2)$$

$$-1 = 2B$$

$$B = -\frac{1}{2}$$

let  $s=3$

$$6-5 = -A$$

$$A = -1$$

let  $s=4$

$$8-5 = 2C$$

$$C = \frac{3}{2}$$

$$\therefore y(s) = \frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)}$$

$$y(t) = \frac{3}{2} e^{4t} - \frac{1}{2} e^{2t} - e^{3t}$$

$$y(t) = \frac{1}{2} (3e^{4t} - e^{2t} - 2e^{3t})$$